Production Research Report No. 20

A Least-Cost BROILER FEED FORMULA

method of derivation

Agricultural Research Service
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PREFACE

The mixing of animal feeds involves two problems. First, the manufacturer or the farmer who mixes the ingredients must have accurate, specific, quantitative information about nutritive requirements. Second, for the most part, he wants to meet these requirements at the lowest possible cost. To solve these problems, animal nutritionists, economists, and statisticians must work together. Animal nutritionists must state all the requirements in terms that can be analyzed objectively. They must also provide information on the chemical and nutritive content of the large number of ingredients that may be considered for a feed mixture. Economists and statisticians must work together to discover the least-cost mixture that will meet all specifications.

The economic problem involved here is formidable. In dealing with a large number of possible ingredients, there may be thousands or even millions of combinations that would meet all nutritive requirements. Economists and statisticians would find it almost impossible to discover all such combinations and price them. Through long experience with differing mixtures of ingredients, feed manufacturers can probably come fairly close to finding the least-cost mixture by trial and error methods. Yet probably, such methods are seldom successful in finding the particular mixture of ingredients that provides the absolute minimum of cost. A manufacturer or a cooperative association handling a large volume of mixed feeds is very much concerned with cost. In fact, a large concern will often find it advantageous to change the feed mixture if it can save only a few cents a ton.

Fortunately, mathematicians have recently found methods of analyzing problems of this kind, however large they may be. These methods are now commonly called "linear programming." The study reported here applies methods of linear programming to the rather complicated problem of mixed feeds. In this instance, we are dealing with feeds for broilers. The mixture must meet a long and detailed list of specifications.

The study reported here was of interest to both the Agricultural Marketing Service and the Agricultural Research Service of the United States Department of Agriculture. Informal study was begun in 1956. Participants included Gordon A. King, Malcolm Clough, and Frederick V. Waugh of the Agricultural Economics Division, AMS, and Ronald L. Mighell of the Farm Economics Research Division, ARS. These men also worked closely with Dr. Charles A. Denton and his staff in the Nutrition Section of the Poultry Research Branch of ARS at Beltsville, Md.

The poultry nutrition specialists provided information and sources of data on feed requirements and composition of feedstuffs. By the end of 1956, Mr. King had made substantial progress in bringing together the data needed for an economic analysis.

Without specific knowledge of the work being done in the United States Department of Agriculture, a project that covered the same general area was planned at the Pennsylvania Agricultural Experiment

Station in the fall of 1956. When the common interest became known early in 1957, it was agreed that a joint research effort was desirable. Under this joint project, the information assembled by Gordon A. King was reviewed and revised. At several points, further judgment on nutritional questions was needed in choosing between sources of information. Robert V. Boucher assumed this responsibility as an active participant in the project.

A linear programming model of the feed problem was then developed by Gordon King and Robert Hutton, with the assistance of Frederick V. Waugh. This model was checked against the practices common in the feed-mixing trade. A company in Lancaster, Pa., was particularly helpful in this connection.

The routine of solving the problems and the preparation of a first draft of this report was handled by Robert Hutton in close consultation with Robert Boucher. This first draft was subsequently revised to reflect the comments and criticisms of the many participants. Richard J. Foote, then of the Agricultural Marketing Service, made a substantial contribution in reviewing an early draft of this report.

This report was authorized for publication on December 4, 1957, as paper number 2212 in the journal series of the Pennsylvania Agricultural Experiment Station.

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A LEAST-COST BROILER FEED FORMULA

Method of Derivation

Robert F. Hutton, Gordon A. King, and Robert V. Boucher[1]

HIGHLIGHTS

This report describes the linear programming model used in deriving a least-cost broiler-feed formula that will meet each of a number of stated specifications. The reasoning back of the development and application of the formula is given.

The report will be useful to feed manufacturers, livestock and poultry producers, and all those concerned with mixing ingredients to produce economical rations.

The important nutrient specifications of broiler rations relate to levels of protein, fiber, productive energy, amino acids, minerals, vitamins, and several unidentified factors. In addition, rations usually contain small amounts of antibiotics and antioxidants.

The text outlines a set of nutritive and other specifications of broiler feeds, including restrictions as to minimum and maximum amounts of the several ingredients that make up the ration.

Composition tables bring together the latest available information on the nutrient composition of the feedstuffs considered in the analysis.

The detailed analysis develops a solution for a realistic broiler feed problem under actual price conditions for a given time and place. Methods are outlined for adjusting the formula to changes in prices, changes in ration specifications, and changes in composition of ingredients.

Certain of the specifications can be met from only one source; an example of this is salt. For certain others the least-cost source can be predetermined. The total problem is simplified by adding the ingredients in these two categories in fixed amounts and eliminating them from the main analysis.

THE BACKGROUND

Present-day broiler feeds are formulated with a complex set of specifications which provides feed that insures rapid growth and high conversion efficiency. Comparison of these feeds with those of 10 years ago indicates the increasing complexity of nutritive and other specifications, and reflects the advances made in the field of poultry nutrition. Also, new ingredients, such as synthetic methionine, have been developed, and specifications for feeds can be met by a greater number of combinations of feed ingredients than was previously possible. Therefore, feed mixers have found it increasingly difficult to take advantage of changes in the prices of ingredients to meet the specifications of a particular feed at minimum cost. The recently developed techniques of linear programming are suited to the study of the economic aspects of this problem, as illustrated in this report.

The rapid expansion of the broiler industry is familiar. The number of commercial broilers produced in 1956 was 1,345 million, as compared with 293 million in 1946 and 53 million in 1936. A large proportion of these broilers are produced on feeds prepared by feed manufacturers, and production of broiler feeds trended sharply upward during this period. As feed costs account for a large percentage of variable costs to producers, finding a minimum-cost feed that will meet given specifications is a continuous problem. Broiler producers must also decide what type of feed will return the highest profit under varying prices for both feed and broiler meat. This aspect of the overall problem is admittedly important, but it is not considered in detail in this report.

The minimum cost feed, as considered here, is one that will furnish at least specified quantities of a list of nutritional and other properties at the lowest cost to

[1]Associate Professor of Farm Management, Pennsylvania State University; Lecturer, University of California (formerly Agricultural Economic Statistician, Agricultural Marketing Service, United States Department of Agriculture); and Professor of Agricultural and Biological Chemistry, Pennsylvania State University, respectively.

the buyer of the ration. Stated more specifically, the problem is to find the combination of feed ingredients that will meet all of a predetermined set of specifications at least cost with given prices. That nutritionists, economists, and statisticians need to cooperate in obtaining practical results from a linear programming solution to this problem was recognized before the problem was formulated. This fact is emphasized throughout this report. Mixing and feed-appearance problems faced by mill operators introduce added complications, a few of which are included in the present discussion.

The report outlines the nutritive and other specifications of broiler feeds, the composition of feed ingredients, and the restrictions as to the amounts of several of these ingredients that can be included in the feed. The concept of linear programming as applied to the problem of minimum-cost feed formulas is outlined, and a detailed explanation is given of the solution under given price conditions. Further, methods are outlined for adapting the results to alternative price situations, feed specifications, and composition of ingredients.

SPECIFICATIONS OF BROILER FEEDS

The important nutrient specifications of broiler feeds relate to the levels of protein, fiber, productive energy, amino acids, minerals, vitamins, and several "unidentified factors." Also, the level of productive energy and the protein content should be in balance. In addition, feeds usually contain small amounts of antibiotics and antioxidants. These factors are included in the broiler feeds at levels considered adequate by poultry nutritionists for so-called "high efficiency" broiler feed. The discussion that follows relates to the specifications for a particular feed that is the basis for further adjustments presented in later sections of this report.

The minimum level of productive energy is taken as 950 calories per pound. The ratio of productive energy (in calories per pound) to the protein content (in percentage) is allowed to vary from 42:1 to 45:1. Assuming that the productive energy specification of 950 calories per pound is just met in the solution to the problem, the protein content would have to fall between 21.11 percent and 22.62 percent, the limits associated with the ratios of 45:1 and 42:1, respectively. However, if the minimum-cost feed contained a productive energy content of 1,050 calories per pound, for example, then the protein content would need to vary between the limits of 23.33 percent and 25.00 percent. Thus, the minimum protein content consistent with the ratio and productive energy specifications is 21.11 percent. This minimum can vary, however, depending on the level of productive energy that enters into the minimum-cost solution.

The specifications for amino acids include those for arginine, lysine, methionine, methionine plus cystine, and tryptophan. These requirements are adapted from those given on page 6 of the National Research Council publication 301.[2] There are at least three viewpoints as to the required minimum level of the amino acids in the event that productive energy rises above the minimum level. These are that: (1) The stated minima applies throughout the range of variation in productive energy and the associated changes in total protein content; (2) the ratio of each of the amino acids to the total protein of the formula remains constant throughout all changes in protein level; or (3) each amino acid should be maintained at a prescribed minimum level per unit of productive energy. Nutritional research on this question is inadequate to support any firm position favoring one of these viewpoints over the others. However, there is some evidence that the amount of feed eaten by the broiler is proportional to the energy of the feed. Within the limit of inherited capacity, the potential for growth is proportional to energy intake and the requirement for amino acids is proportional to growth. This rather tenuous chain of inferences was used in deciding on a constant ratio of each amino acid to productive energy in this problem.

The specification for fiber places a maximum level of 4 percent as acceptable for this feed.

The specification for calcium requires that the broiler feed contain exactly 1.1 percent of this mineral. Similarly, the

[2]National Research Council, Committee on Animal Nutrition, Subcommittee on Poultry Nutrition, Nutrient Requirements for Domestic Animals: 1. Nutrient Requirements for Poultry. Natl. Res. Counc. Pub. 301, 27 pp., illus. 1954.

inorganic (available) phosphorus content must be exactly 0.45 percent. Again, the nutritional evidence in support of the exact levels used in these specifications is inadequate. It is known that some variation in the level is feasible from a nutritional viewpoint. It is known also that excessive intakes, as well as deficiencies, of minerals may be harmful. The exact levels are used here as the simplest assumption that could be made concerning requirements for these minerals. Later in the report, a procedure to be used in relaxing this assumption is presented.

The specification for added salt is 5 pounds per ton. A level of 10 pounds per ton is commonly used, and there is no nutritional evidence for choice of one level over the other.

In addition to calcium, phosphorus and salt, requirements are established for such elements as potassium, manganese, iodine, and magnesium. Of the latter, only iodine and manganese are of concern, as the required levels of the others are normally met in feed mixtures. The levels of specification of iodine and manganese are not given here as feed manufacturers almost universally meet these requirements by adding a prepared mineral supplement.

The tabulation that follows summarizes the specifications for the characteristics of the formula:

Characteristic	Level
Productive energy per pound------calorie--	950 or more
Protein------percent--	21.11 or more
Ratio of energy to protein------calories per pound to percentage--	42:1 to 45:1
Nonfiber------percent--	96 or more
Arginine------do----	1.2 or more
Lysine------do----	.9 or more
Methionine------do----	.48 or more
Methionine plus cystine------do----	.8 or more
Tryptophan------do----	.2 or more
Calcium------do----	1.1 exactly
Inorganic phosphorus------do----	.45 exactly
Added salt------do----	.25 exactly
Vitamins per ton:	
Riboflavin------gram--	3.1 or more
Pantothenic acid------do---	10.0 or more
Niacin------do---	28.8 or more
Choline------do---	1440. or more
Folic acid------do---	.6 or more
B_{12}------milligram--	9.6 or more
A------million I.U.--	4 or more
D------thousand I.C.U.--	270 or more
K------milligram--	432 or more

Some additional requirements for broiler feed, which include the so-called "unidentified growth factors," are not given in this tabulation. These requirements were not included because the accuracy with which they were established is lower than that of the other characteristics.

Sources of the "unidentified factors" are specified in amounts that are considered adequate now. The fact that they are unidentified indicates that the specifications should be considered as approximations until more thorough nutritional findings are completed. According to a report of the Agricultural Research Service,[3] these factors are thought to be present in the following ingredients: Fish factor--fishmeal, fish solubles, crabmeal, meat and poultry byproducts, liver preparations, and certain fermentation products; alfalfa factor--dehydrated alfalfa leaf meal, grass juice concentrate, and dried brewers' yeast; and whey factor--

[3] U.S. Agricultural Research Service. Ingredients in the modern Broiler Diet. U.S. Agr. Res. Serv. ARS 22-25, 10 pp. 1956.

dried distillers' solubles, dried distillers' molasses solubles, dried brewers' yeast, butyl fermentation solubles, dried whey products, and certain fermentation products. The report indicates that several other factors are believed to exist, but only limited information concerning them is available at present.

In addition to the unidentified growth factors, most broiler feeds are formulated to contain a pigmentation factor. This factor in the feed imparts a yellow color to the skin of the dressed broiler. Several recently developed sources of this pigmentation factor include some chemicals. However, in the feed formulated in the study reported here, only corn gluten meal at the 3-percent level was added specifically to meet this requirement, although alfalfa meal and yellow cornmeal also contribute to pigmentation. The corn gluten meal produces consistently reliable results, and it is in general use throughout the feed manufacturing trade.

Small amounts of antibiotics are normally included in a broiler feed. The specification for the formula developed in the study reported here includes 4 grams of penicillin.

One-tenth of a pound of 3-nitro-4-hydroxyphenylarsonic acid, a growth-stimulating compound, is included in the specifications of this formula.

An antioxidant, B.H.T., is included at the 0.25 pound per ton level. This antioxidant retards development of rancidity in fats, particularly added fats such as tallow, and lessens the loss during storage of fat-soluble vitamins. This ingredient is sometimes omitted from broiler formula specifications if the period of storage is to be short.

Specifications for vitamins include riboflavin, pantothenic acid, niacin, choline, folic acid, and vitamins A, D, B_{12}, and K. The specifications were adapted from those indicated in National Research Council Publication 301;[4] they include a safety margin of 66 percent for vitamin A, 50 percent for vitamin D, and 20 percent for the water-soluble vitamins and vitamin K.

Feed manufacturers commonly add a vitamin supplement without close regard to the vitamin content of the feed ingredients. This is justified on the grounds that the vitamin content of an ingredient varies considerably from its average level and that the consequences of feeding a ration low in vitamin content may be so serious that the risk is not worth taking. The total allowances for vitamins given in the tabulation on page 3 would amount to less than $3.00 per ton of broiler feed when bought as vitamin supplements. Thus, the potential saving by economizing on vitamins is small relative to the loss in feed efficiency that would attend a vitamin deficiency. On page 34 of the appendix, a procedure is outlined for use in economizing on vitamins, accepting the average values for vitamin content of ingredients. The discussion in the main body of the report presumes the use of vitamin supplements.

Certain of the specifications listed in the tabulation on page 3 can be met from only one source. The least-cost source for other requirements can be predetermined. The analysis can be simplified by beginning with the assumption that ingredients of this kind are added to the mix in fixed amounts. A list of these ingredients and the amount of each added are given in the tabulation on the following page, together with the vitamin supplements added. An error made in selecting a source of one of the factors would be revealed in the subsequent analysis.

COMPOSITION OF FEED INGREDIENTS

Table 1 indicates the composition of important ingredients used in broiler feeds. The list would differ somewhat for certain regions of the country and for specific lots of the indicated ingredients, which vary to some extent in content of the indicated nutrients. In general, the composition of the byproduct feeds is as given in National Research Council Publication 449 but, unless otherwise indicated, the values for productive energy content for these feeds and for the feed grains are those given in Titus.[5] In most instances, the composition of the feed grains is as given in National Research Council Publication 301.[6] Footnotes to the table indicate partially the difficulty in obtaining comparable data on these feeds.

[4] See footnote 2, p. 2.

[5] National Research Council, Committee on Feed Composition, Composition of Concentrate By-Product Feeding Stuffs, Natl. Res. Counc. Pub. 449, 126 pp., 1956; Titus, H. W., The Scientific Feeding of Chickens, rev. of 2d ed., with addendum, 297 pp., illus., Danville, Ill., 1955.

[6] See footnote 2, page 2.

Item	Amount
Salt----pound--	5.00
Mineral supplement----do---	6.00
Corn gluten meal----do---	60.00
Dried corn distiller's solubles----do---	80.00
Fish solubles, condensed----do---	20.00
Alfalfa meal, dehydrated, 17 percent----do---	40.00
3-Nitro-4-hydroxyphenylarsonic acid----do---	.10
B.H.T.----do---	.25
Penicillin----gram--	4.00
Vitamin:	
Riboflavin----do---	4.00
Calcium pantothenic----do---	4.00
Niacin----do---	20.00
Choline chloride----do---	500.00
Folic acid----do---	1.00
K----do---	1.00
B_{12}----milligram--	6.00
A----million I.U.--	4.00
D----million I.C.U.--	1.50
Weight of above items----pound--	212.58

Certain ingredients that provide for only one or two nutritional specifications are omitted from table 1. These include dicalcium phosphate, containing 27 percent calcium and 19.07 percent inorganic phosphorus; defluorinated phosphate, containing 27.18 percent of calcium and 13.34 percent of inorganic phosphorous; calcium carbonate, containing 36.57 percent calcium, and synthetic methionine, containing 100 percent methionine. As noted above, salt and a trace mineral supplement are also included as ingredients in the formula.

Usually, vitamins and synthetic methionine are added on a carrier. Here, however, it is assumed that these items are available, as straight materials. This assumption is valid in general in that the mixer has considerable latitude in specifying the carrier.

Certain of the ingredients in table 1 cannot be used in unlimited amounts. The maximum limits for these restricted ingredients follow:

Item	Maximum per ton
	Pounds
Meat scrap----	240
Meat and bone scrap----	240
Meat scrap plus meat and bone scrap----	240
Fish-	
Meal, Menhaden----	300
Solubles, condensed----	100
Dried-	
Buttermilk----	200
Skim milk----	200
Whey, cheese----	200
Buttermilk plus skim milk plus whey cheese----	200
Wheat, red dog flour----	200
Tallow:	
Pelleted feeds----	60
Unpelleted feeds----	160

Each of the other ingredients may enter the formula in any amount so long as the specifications are met.

TABLE 1.--COMPOSITION OF INGREDIENTS OF BROILER-FEED FORMULA[1]

(Percentages or units per ton)

Ingredient	Protein	Non-fiber	Productive energy[2]	Amino acids: Arginine	Amino acids: Lysine	Amino acids: Methionine	Amino acids: Methionine plus cystine	Amino acids: Tryptophan	Minerals: Calcium	Minerals: Inorganic phosphorus[3]
	Percent	*Percent*	*1,000 calories*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
Soybean meal:										
Solvent, 44 percent	45.8	94.2	1,520	3.20	2.90	0.62	1.28	0.60	0.32	0.20
Without hulls, 50 percent	50.9	97.2	1,580	[4] 3.58	[4] 3.58	[4] .75	[4] 1.49	[4] .65	.26	.19
Corn gluten meal, 41 percent	42.9	96.0	1,642	1.4	.8	1.0	1.6	.2	.16	.12
Meat scrap[5]	53.4	97.6	1,898	3.7	3.8	.8	1.4	.3	7.94	4.03
Meat and bone scrap[5]	50.6	97.8	1,748	4.0	3.5	.7	1.3	.2	10.57	5.07
Fish-										
Meal, Menhaden[6]	61.3	99.3	1,882	4.0	5.3	[7] 1.80	[7] 2.8	.6	5.49	2.81
Solubles, condensed[8]	31.4	99.4	[9] 880	2.4	2.7	1.0	2.7	.8	.61	.70
Dried-										
Buttermilk	32.0	99.6	1,572	1.1	2.4	.7	[10] 1.0	.5	1.34	.94
Skim milk	33.5	99.8	1,530	1.2	2.8	.8	1.3	.4	1.26	1.03
Whey, cheese[11]	13.1	99.7	1,572	.2	.8	.15	.46	.1	.9	.80
Brewers' yeast	44.6	97.3	1,144	2.2	3.0	.7	1.2	.5	.13	.43
Corn distillers' solubles	26.9	96.2	2,040	1.0	.9	.6	1.2	.2	.35	.40
Corn distillers' grains, with solubles	27.2	91.0	1,782	.9	.7	.5	[12] .8	.1	.17	.20
Alfalfa meal, dehydrated, 17 percent[13]	17.8	75.8	[14] 522	.8	.9	.32	.66	.23	1.7	.06
Cornmeal, yellow	8.9	98.0	2,210	.4	.3	.14	.29	.08	.02	.09
Milo, maize	11.3	97.8	2,198	.3	.3	.16	.36	.09	.03	.09
Wheat:										
Hard red winter	15.2	97.4	1,794	.5	.4	.21	.45	.16	.05	.12
Soft red winter[15]	10.2	97.9	1,794	.3	.3	.14	.3	.11	.04	.09
Standard middlings	17.2	92.4	1,368	.9	.7	.2	.4	.2	.15	.27
Red dog flour[16]	16.0	97.0	[9] 2,000	1.0	.6	.1	.3	.2	.10	.15
Hominy feed, yellow	11.1	95.1	1,670	.5	.4	.1	[4] .23	.1	.05	.16
Oats:										
Excluding Pacific Coast[17]	12.0	89.0	1,620	.6	.4	.13	.35	.14	.09	.12
Feeding, rolled[18]	15.0	98.0	2,324	[4] .96	[4] .61	[4] .24	[19] .51	[4] .21	.07	.12
Barley, excluding Pacific Coast[20]	12.7	94.6	1,626	.5	.3	.12	.32	.13	.09	[21] .12
Tallow	0	100.0	5,756	0	0	0	0	0	0	0
Bonemeal, steamed	12.1	98.3	[22] 600	0	0	0	0	0	28.98	13.59

Ingredient	Vitamins[23]						
	Riboflavin	Pantothenic Acid	Niacin	Choline	Folic Acid	A	B_{12}
	Grams	*Grams*	*Grams*	*Grams*	*Grams*	*Million I.U.*	*Milligrams*
Soybean meal:							
Solvent, 44 percent	3.0	13.2	24.2	2,494	.54	0	0
Without hulls, 50 percent	2.8	13.2	19.6	2,510	.54	0	0
Corn gluten meal, 41 percent	1.4	9.4	45.4	300	.20	24.7	0
Meat scrap[5]	4.8	4.4	51.6	1,774	1.48	0	46.4
Meat and bone scrap[5]	4.0	3.4	43.4	1,986	1.48	0	89.6
Fish-							
Meal, Menhaden[6]	4.4	8.2	50.8	3,326	2.17	0	69.6
Solubles, condensed[8]	13.2	32.2	153.4	3,662	.16	2.0	599.2
Dried-							
Buttermilk	28.2	27.4	7.8	1,644	.36	0	16.6
Skim milk	18.2	30.6	10.4	1,294	.56	0	38.0
Whey, cheese[11]	16.2	44.8	10.2	1,828	.80	0	15.2
Brewers' yeast	31.8	99.8	406.8	3,532	8.80	0	0
Corn distillers' solubles	15.4	19.0	104.8	4,380	1.02	1.0	0
Corn distillers' grains, with solubles	7.8	10.0	60.8	2,246	.74	5.7	0
Alfalfa meal, dehydrated, 17 percent[13]	14.6	24.6	17.4	800	7.86	200.0	0
Cornmeal, yellow	1.0	5.2	9.6	400	.2	4.4	0
Milo, maize	.3	.8	26.2	500	.2	0	0
Wheat:							
Hard red winter	1.0	12.6	48.2	900	.4	0	0
Soft red winter[15]	1.0	12.6	48.2	900	.4	0	0
Standard middlings	1.8	18.0	89.6	976	.82	0	0
Red dog flour[16]	1.4	12.4	47.8	976	.82	0	0
Hominy feed, yellow	2.2	7.8	39.2	870	.30	10.3	0
Oats:							
Excluding Pacific Coast[17]	.8	13.6	16.4	900	.2	0	0
Feeding, rolled[18]	1.2	13.2	9.0	1,010	.54	0	0
Barley, excluding Pacific Coast[20]	1.6	7.4	48.2	1,000	.54	0	0
Tallow	0	0	0	0	0	0	0
Bonemeal, steamed	.8	2.2	3.8	0	0	0	0

See footnotes at end of table.

-Continued

TABLE 1.--COMPOSITION OF INGREDIENTS OF BROILER-FEED FORMULA[1]--Continued

[1] Unless otherwise noted, values are from Natl. Res. Counc. Pubs. 301 and 449. (See footnote 2, p. 2, and footnote 5, p. 4.)

[2] Unless otherwise noted, from Titus, The Scientific Feeding of Chickens. (See footnote 5, p. 4.)

[3] The inorganic phosphorus content of ingredients from plants is taken as 30 percent of total phosphorus.

[4] Unpublished list of analytical values for feed ingredients, 1957, prepared by the Eastern State Farmers' Exchange.

[5] Value for tankage used for folic acid.

[6] Values for productive energy, pantothenic acid, and choline content not specified for Menhaden--they refer to "fishmeal"; that for cystine is for "fishmeal, 67 percent protein"; and that for folic acid is for "fishmeal, herring."

[7] As shown in Natl. Res. Counc. Pub. 301.

[8] Value for folic acid is for "solubles, condensed, tuna."

[9] Adapted from Hubbell, C. H., Feedstuffs--1957 Feedstuffs Analysis Table, Feedstuffs 29(12):42, 1957.

[10] As shown in The Feed Bag Red Book . . . Buyers' Guide for 1956, 268 pp., Editorial Service Company, Milwaukee, Wis.

[11] Values for methionine, cystine, calcium, phosphorus, pantothenic acid, and niacin content are for "whey, dried", as indicated in Natl. Res. Counc. Pub. 301.

[12] Calculated by assuming that the cystine content is 1.1 percent of the protein content, a factor for corn given in Block, R. J., and Bolling, Diana, The Amino Acid Composition of Proteins and Foods; Analytical Methods and Results, 398 pp., illus., Springfield, Ill., 1945.

[13] Meal with guaranteed vitamin A content of 100,000 I.U. per pound.

[14] Calculated from Fraps, G. S., Composition and Productive Energy of Poultry Feeds and Rations, Tex. Agr. Expt. Sta. Bul. 678, 37 pp., 1946.

[15] Values for protein and fiber from Morrison, F. B., Feeds and Feeding, ed. 21, unabridged, 1,207 pp., illus., Ithaca, N. Y., 1948. Values for vitamins and amino acids are as shown in Natl. Res. Counc. Pub. 301, with the amino acids scaled down relative to protein content.

[16] Value for cystine, choline, and folic acid assumed to equal those for wheat, standard middlings.

[17] Value for productive energy is for "all oats."

[18] Values are as shown in Natl. Res. Counc. Pub. 301, except where noted. Productive energy value refers to "oatmeal or groats."

[19] Calculated by assuming that the cystine content is 1.8 percent of the protein content, a factor for oats given in Block and Bolling.

[20] Value for productive energy is for "all barley."

[21] As shown in Morrison.

[22] Value is for "bonemeal, special steamed, 8.6 percent protein."

[23] Values for vitamins were taken from or based on one or more of the following sources: Morrison; The Feed Bag Red Book; Natl. Res. Counc. Pub. 301; Natl. Res. Counc. Pub. 449.

SELECTION OF LEAST-COST FORMULA

A linear programming model of a broiler feed-mixing problem was developed. This model reflects the specifications, restrictions, and composition of ingredients previously discussed. Not all of the relevant restrictions are taken directly into account within the model, but provision is made for handling all restrictions and specifications.

This model is a general one. That is, it is designed to be used without structural change for all possible ranges in the price of ingredients. In addition, it can be modified readily to deal with variations in restrictions, specifications, and composition of ingredients. (For details on the model, see appendix, page 25.)

To demonstrate the usefulness of this model, it has been used to develop a formula. The prices used are presented in the tabulation on page 32; they were realistic for the Philadelphia, Pa., area around April 1, 1957.

The least-cost formula for the given specifications and the given prices for ingredients is presented in table 2. The cost of this formula is $70.95 per ton. Theoretically, with the ingredients and prices used, it is impossible to obtain a less expensive formula that will satisfy the specifications.

The nutritional composition of this formula, together with the specifications, are presented in table 3. The cost of the formula would have been increased had the specifications other than those for vitamins, which are exceeded, been met exactly.

DISCUSSION OF THE FORMULA SOLUTION

The formula presented in table 2 is somewhat unusual according to trade standards. It contains dried corn distillers' grains with solubles. This is not a commonly used ingredient in broiler feeds, and especially not in a starter. Also, by trade standards, the formula contains a high level of synthetic methionine. One possible point of confusion concerning the level of synthetic methionine may benefit from discussion. That is, the high level of synthetic methionine used is economical, even though its cost is in excess of $2.60 per pound. If any synthetic methionine is removed from the formula, it must be replaced by a combination of other ingredients and the cost is increased.

TABLE 2.--LEAST-COST FORMULA FOR TON OF BROILER FEED AT GIVEN PRICES[1]

Ingredient	Amount of formula	Percentage	Cost
	Pounds	*Percent*	*Dollars*
Soybean meal, solvent, 44 percent	345.98	17.299	10.41
Corn gluten meal, 41 percent	112.22	5.611	4.51
Meat and bone scrap	125.46	6.273	3.76
Fish solubles, condensed	20.00	1.000	.92
Dried corn distillers' solubles	80.00	4.000	3.94
Dried corn distillers' grains with solubles	199.22	9.961	6.30
Alfalfa meal, dehydrated, 17 percent	40.00	2.000	1.62
Cornmeal, yellow	1,047.00	52.350	30.57
Calcium, carbonate	15.36	.768	.10
Salt	5.00	.250	.04
Mineral supplement	6.00	.300	.18
Methionine	2.18	.109	5.73
Vitamins, 3-Nitro-4-hydroxyphenylarsonic acid, B.H.T., and penicillin[2]	1.58	.079	2.87
Total	2,000.00	100.000	70.95

[1] Prices are tabulated on page 10.
[2] For amount of each item included in this group, refer to the tabulation on page 5.

Ingredient	Price
Soybean meal:	*Dollars*
Solvent, 44 percent----ton--	60.20
Without hulls, 50 percent----do--	67.20
Corn gluten meal, 41 percent----do--	80.40
Meat scrap----do--	76.60
Meat and bone scrap----do--	60.00
Fish-	
Meal, Menhaden----do--	150.60
Solubles, condensed----do--	92.10
Dried-	
Buttermilk----do--	325.40
Skim milk----do--	225.20
Whey, cheese----do--	123.00
Brewers' yeast----do--	210.00
Corn distillers'-	
Solubles----do--	98.50
Grains, with solubles----do--	63.20
Alfalfa meal, dehydrated, 17 percent----do--	80.80
Cornmeal, yellow----do--	58.40
Milo, maize----do--	65.00
Wheat:	
Hard red winter----do--	95.00
Soft red winter----do--	79.80
Standard middlings----do--	55.20
Red dog flour----do--	55.90
Hominy feed, yellow----do--	54.20
Oats:	
Excluding Pacific Coast----do--	62.20
Feeding, rolled----do--	97.60
Barley, excluding Pacific Coast----do--	56.20
Tallow----do--	147.20
Bonemeal, steamed----do--	80.00
Calcium, carbonate----do--	13.60
Phosphate:	
Dicalcium----do--	86.00
Defluorinated----do--	91.60
Salt----do--	17.40
Mineral supplement----do--	60.00
Methionine----pound--	2.63
Antioxidant (B.H.T.)----do---	1.28
3-Nitro-4-hydroxyphenylarsonic acid----do---	4.45
Penicillin----gram--	.07
Vitamin:	
A[1]----million I.U.--	.14
D[2]----million I.C.U.--	.03
K----kilogram--	70.00
B_{12}----gram--	30.00
Calcium pantothenate[3]----kilogram--	32.00
Niacin----do----	8.00
Choline chloride[4]----do----	.94
Folic acid----do----	30.00
Riboflavin----do----	45.00

[1] Vitamin A concentration of 250,000 I.U. per gram. [2] Vitamin D concentration of 200,000 I.C.U. per gram. [3] Calcium salt of pantothenic acid. This material is assumed to be equivalent to pantothenic acid, although there is a slight difference in molecular weight. [4] This salt of choline is assumed to be equivalent to choline although there is a slight difference in molecular weight..

TABLE 3.--NUTRITIONAL COMPOSITION OF LEAST-COST FORMULA AT GIVEN PRICES COMPARED WITH SPECIFICATIONS
(Percentages or units per ton)

Characteristic	Level in-- Formula	Level in-- Specification	Amount in formula in excess of specification
Weight--pound--	2000	2000	None
Productive energy per pound--calories--	950	950 or more	None
Protein--percent--	22.6190	21.11 or more	1.51
Ratio of calories per pound to percentage of protein--	42:1	42:1 to 45:1	None
Nonfiber--percent--	96.0	96 or more	None
Arginine--do--	1.3	1.2 or more	.1
Lysine--do--	1.1	0.9 or more	.2
Methionine--do--	0.48	0.48 or more	None
Methionine plus cystine--do--	0.3	0.8 or more	None
Tryptophan--do--	0.2	0.2 or more	None
Calcium--do--	1.1	1.1 exactly	None
Inorganic phosphorus--do--	.45	.45 exactly	None
Fish factor:			
Fishmeal, Menhaden *or*--pound--	0	[1] 50	[2] None
Fish solubles, condensed--do--	20	[1] 20	[2] None
Alfalfa factor:			
Alfalfa meal, dehydrated, 17 percent--do--	40	[1] 40	None
Whey factor:			
Dried whey, cheese *or*--do--	0	[1] 40	[3] None
Dried corn distillers' solubles, *or*--do--	80	[1] 80	[3] None
Dried brewers' yeast--do--	0	[1] 40	[3] None
Pigmenting factor:			
Corn gluten meal, 41 percent--do--	112.22	60 or more	52.22
Vitamins:			
A--million I.U.--	12.3	4.0 or more	8.3
D--million I.C.U.--	1.5	.275 or more	1.225
K--gram--	[4] 1.0	.432 or more	[4] .568
B_{12}--milligram--	17.6	9.6 or more	8.0
Pantothenic acid--gram--	12.3	10.0 or more	2.3
Niacin--do--	46.6	28.8 or more	17.8
Choline--do--	1733.8	1440.0 or more	293.8
Folic acid--do--	1.6	.6 or more	1.0
Riboflavin--do--	7.2	3.1 or more	4.1

[1] Any single ingredient at level listed in specification or a linear combination of two or more of the ingredients will meet the "unknown growth factor" indicated.
[2] 20 pounds condensed fish solubles meet the fish factor.
[3] 80 pounds dried corn distillers' solubles meet the whey factor.
[4] Possible vitamin content of feed ingredients is not included because of the lack of reliable data on analysis.

ADJUSTMENT OF THE FORMULA TO MEET ADDITIONAL SPECIFICATIONS

The discussion of the least-cost feed formula suggests the need for additional specifications. For example, a feed mixer may wish to avoid use of "dried corn distillers' grains, with solubles," in the absence of further proof of its value in broiler feeds. Or, he may insist that synthetic methionine be held to a lower level in the absence of proof that higher levels are nutritionally sound.

Additional specifications may be needed for other reasons. For example, the handling properties of condensed fish solubles may make them unsuitable for use by some mixers. Or, a limited supply of a certain ingredient may call for restriction in its use below the least-cost amount.

Table 4 was prepared to cover situations similar to those presented above, and thereby to extend the usefulness of the solution. Among other data given in table 4 are the displacement rates between ingredients not in the solution and those in the solution. Refer, for illustration, to column 22 of table 4. The upper or ingredient part of this column shows the changes that would need to be made if 1 percent of "wheat standard middlings" were added to the formula. The middle or nutritional characteristic section of this column shows the changes in computed analysis that would result if 1 percent of wheat standard middlings were added to the formula and the changes indicated in the ingredient section of the column were made. The cost section of the column indicates the increase in cost that would result if 1 percent of wheat standard middlings were added. The change in cost is, of course, specific to the set of prices used in the problem. (See page 10). The changes in ingredients and nutritional characteristics indicated in this column are equivalent in nutritional terms to 1 percent of wheat standard middlings.

The combination of adjustments is necessarily nutritionally equivalent only with respect to the characteristics for which the specifications are given. It may not be equivalent, for instance, for bulk or any other characteristic that is not considered in the set of specifications used for this feed.

Displacing an Ingredient from the Formula

With the information given in table 4 for the various ingredients, the solution presented in table 2 can be adjusted with facility. For example, the problem of displacing dried corn distillers' grains with solubles can be attacked. Column 17 of table 4 represents the changes required to increase the amount of dried corn distillers' grains with solubles. If a reduction is desired, all that is necessary is to change the signs on all the numbers given in column 17. Presuming that complete displacement of the dried corn distillers' grains with solubles is desired, this would mean that 9.9610 percent must be displaced. The product of the changes indicated by column 17 and 9.9610 indicate the needed adjustments for the complete displacement of dried corn distillers' grains with solubles.

In following through this adjustment, an immediate problem arises. One change called for is a displacement of soybean meal without hulls. However, there is no soybean meal without hulls in the solution given in table 2. This means that this adjustment cannot be made alone. Some other change that will increase the amount of soybean meal without hulls must be made simultaneously with the reduction in the amount of dried corn distillers' grains with solubles.

Examination of table 4 indicates many possible sources of increase in amount of soybean meal without hulls. Altogether, the amount of this ingredient may be increased in 23 ways, using single changes as presented in table 4. The problem becomes one of choosing the "best" way. At this point the best and the least-cost ways are considered to be synonymous. If this leads to an unsatisfactory formula, some revision may be needed.

The technique for appraising the effect on cost of the several ways of increasing the amount of soybean meal without hulls in the formula is easy. All changes indicated by each of the 23 figure columns in table 4 that increase the amount of this ingredient can be ignored, except the changes in the amount of soybean meal without hulls itself and the changes in cost. By dividing the latter by the former, a relative cost for the various sources of increase of soybean meal without hulls can be obtained. All else being equal, the lowest cost source is chosen.

TABLE 4.--INGREDIENT DISPLACEMENT RATES AND ACCOMPANYING CHANGES IN FORMULA COMPOSITION TO REACH NEW LEAST-COST FORMULAS WITH ALTERNATIVE CHANGES IN SPECIFICATIONS[1]

Ingredient or characteristic	Displacement rates and accompanying changes per ton of feed from increasing--						
	Costless inert filler, 1.0 percent	Protein, 1.0 percent	Productive energy, 50 cal. per pound	Methionine, 0.1 percent	Tryptophan, 0.1 percent	Calcium, 0.1 percent	Inorganic phosphorus, 0.1 percent
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ingredient:							
Soybean meal:							
Solvent, 44 percent------------ton--	-0.168448	-0.118808	-0.721671	-0.016862	0.092518	-0.046033	0.129759
Without hulls, 50 percent--------do--	.156918	.101423	.667176	.015704	.151542	.042872	-.119309
Corn gluten meal, 41 percent-------do--	-.009333	.032525	-.044224	-.000933	-.283131	-.002551	-.019006
Meat and bone scrap----------------do--	.000809	.000387	.002821	.000081	-.003357	.000221	.019622
Cornmeal, yellow-------------------do--	.009989	-.015268	.095241	.001000	.041040	.002729	-.005345
Calcium, carbonate-----------------do--	.000160	.000073	.000897	.000016	.000299	.002777	-.005869
Methionine-------------------------do--	-.000059	-.000331	-.000240	.000994	.001087	-.000016	.000150
Nutritional characteristic:							
Productive energy per pound---calorie--	0	0	50.000000	0	0	0	0
Protein---------------------percent--	0	1.000000	0	0	0	0	0
Nonfiber----------------------do-----	.553412	.304687	2.297798	.055382	.096909	.151204	-.374982
Arginine----------------------do-----	.016782	.023885	.066613	.001681	.445192	.004584	.037841
Lysine------------------------do-----	.071551	.041345	.298868	.007160	.584941	.020030	.000008
Methionine plus cystine---------do-----	.001304	.014061	.006854	.100132	.007470	.000357	-.003080
Methionine--------------------do-----	0	0	0	.100000	0	0	0
Tryptophan--------------------do-----	0	0	0	0	.100000	0	0
Calcium-----------------------do-----	0	0	0	0	0	.100000	0
Inorganic phosphorus------------do-----	0	0	0	0	0	0	.100000
Whey factor-------------------do-----	0	0	0	0	0	0	0
Fish factor-------------------do-----	0	0	0	0	0	0	0
Change in formula cost----------dollar--	-.022744	-.332488	2.313428	5.264278	.914302	.030837	-.158721

See footnote at end of table.

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TABLE 4.--INGREDIENT DISPLACEMENT RATES AND ACCOMPANYING CHANGES IN FORMULA COMPOSITION TO REACH NEW LEAST-COST FORMULAS WITH ALTERNATIVE CHANGES IN SPECIFICATIONS[1]--Continued

Ingredient or characteristic	Displacement rates and accompanying changes per ton of feed from increasing--					
	Meat scrap, 1.0 percent	Fish meal, Menhaden, 1.0 percent	Fish solubles, condensed, 1.0 percent	Dried buttermilk, 1.0 percent	Dried skim milk, 1.0 percent	Dried whey, cheese, 1.0 percent
(1)	(9)	(10)	(11)	(12)	(13)	(14)
Ingredient:						
Soybean meal:						
Solvent, 44 percent-------------ton--	0.014777	0.026468	-0.079661	-0.026491	-0.028181	-0.046384
Without hulls, 50 percent--------do--	-.015647	-.032757	.058401	.016374	.020429	.042687
Corn gluten meal, 41 percent-------do--	-.000054	.002949	.008576	.003561	.000216	-.002047
Meat and bone scrap----------------do--	-.007922	-.005407	-.000687	-.001470	-.001673	-.001242
Cornmeal, yellow-------------------do--	-.001259	-.001215	.003227	-.002082	-.000983	-.003226
Calcium, carbonate-----------------do--	.000101	.000051	.000274	.000159	.000240	.000226
Methionine-------------------------do--	.000003	-.000088	-.000130	-.000051	-.000048	-.000014
Nutritional Characteristic:						
Productive energy per pound---calorie--	0	0	0	0	0	0
Protein----------------------percent--	0	0	0	0	0	0
Nonfiber-----------------------do-----	-.045734	-.066266	.253265	.096956	.109031	.163876
Arginine-----------------------do-----	-.003995	-.010558	.011290	-.016880	-.011828	-.002735
Lysine-------------------------do-----	-.003291	-.004434	.010531	.002902	.013474	.019391
Methionine plus cystine---------do-----	-.000800	.001571	.012858	-.001425	.000496	.001618
Methionine---------------------do-----	0	0	0	0	0	0
Tryptophan---------------------do-----	0	0	0	0	0	0
Calcium------------------------do-----	0	0	0	0	0	0
Inorganic phosphorus------------do-----	0	0	0	0	0	0
Whey factor--------------------do-----	0	0	0	0	0	50.000000
Fish factor--------------------do-----	0	40.000000	100.000000	0	0	0
Change in formula cost----------dollar--	.068085	.277007	.205693	2.569625	1.538350	.808085

Ingredient or characteristic	Displacement rates and accompanying changes per ton of feed from increasing--					
	Dried brewers' yeast, 1.0 percent	Dried corn distillers' solubles, 1.0 percent	Dried corn distillers' grains, with solubles, 1 percent	Alfalfa meal dehydrated, 17%, 1.0 percent	Milo maize, 1.0 percent	Wheat: hard red winter, 1.0 percent
(1)	(15)	(16)	(17)	(18)	(19)	(20)
Ingredient:						
Soybean meal:						
Solvent, 44 percent--------------ton--	-0.041362	0.006280	-0.009458	-0.104180	0.001971	-0.023409
Without hulls, 50 percent--------do--	.031254	-.007172	.009793	.093453	-.001858	.020271
Corn gluten meal, 41 percent-------do--	-.003708	-.002492	-.006998	-.005723	-.000546	-.001553
Meat and bone scrap----------------do--	-.000371	-.000601	-.000165	.000512	-.000003	.000060
Cornmeal, yellow-------------------do--	.003975	-.006096	-.003230	.006325	-.009565	-.005404
Calcium, carbonate-----------------do--	.000225	.000088	.000047	-.000344	-.000004	.000039
Methionine-------------------------do--	-.000014	-.000007	.000011	-.000042	.000005	-.000005
Nutritional characteristic:						
Productive energy per pound---calorie--	0	0	0	0	0	0
Protein----------------------percent--	0	0	0	0	0	0
Nonfiber-----------------------do-----	.133085	-.030860	-.027752	.109692	-.006909	.069900
Arginine-----------------------do-----	-.003553	-.003910	.002045	.005752	-.001046	-.001434
Lysine-------------------------do-----	.018907	-.004373	.007495	.040490	.001252	.006038
Methionine plus cystine---------do-----	-.001022	.002067	-.000777	.001618	.000162	.000245
Methionine---------------------do-----	0	0	0	0	0	0
Tryptophan---------------------do-----	0	0	0	0	0	0
Calcium------------------------do-----	0	0	0	0	0	0
Inorganic phosphorus------------do-----	0	0	0	0	0	0
Whey factor--------------------do-----	50.000000	25.000000	0	0	0	0
Fish factor--------------------do-----	0	0	0	0	0	0
Change in formula cost-----------dollar--	1.551352	.253001	.018125	.530476	.067404	.440329

See footnote at end of table.

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TABLE 4.--INGREDIENT DISPLACEMENT RATES AND ACCOMPANYING CHANGES IN FORMULA COMPOSITION TO REACH NEW LEAST-COST FORMULAS WITH ALTERNATIVE CHANGES IN SPECIFICATIONS[1]--Continued

Ingredient or characteristic	Displacement rates and accompanying changes per ton of feed from increasing--						
	Wheat			Hominy feed, yellow, 1.0 percent	Oats, excluding Pacific Coast, 1.0 percent	Oats, feeding, rolled, 1.0 percent	Barley, excluding Pacific Coast, 1.0 percent
	Soft red winter, 1.0 percent	Standard middlings, 1.0 percent	Red dog flour, 1.0 percent				
(1)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
Ingredient:							
Soybean meal:							
Solvent, 44 percent-----------ton--	-0.031257	-0.053650	-0.008307	-0.037379	-0.039532	0.014283	-0.038195
Without hulls, 50 percent------do--	.028282	.047436	.005427	.034262	.035381	-.015777	.034441
Corn gluten meal, 41 percent-----do--	-.001028	-.002645	.000290	-.002400	-.001845	.002280	-.002330
Meat and bone scrap--------------do--	-.000270	-.000111	-.000047	.000013	.000114	-.000073	.000107
Cornmeal, yellow-----------------do--	-.005861	-.001151	-.007394	-.004570	-.004156	-.010696	-.004064
Calcium, carbonate---------------do--	.000147	.000135	.000023	.000079	.000047	-.000015	.000046
Methionine-----------------------do--	-.000012	-.000014	.000009	-.000005	-.000010	-.000002	-.000005
Nutritional characteristic:							
Productive energy per pound-calorie--	0	0	0	0	0	0	0
Protein--------------------percent--	0	0	0	0	0	0	0
Nonfiber---------------------do-----	.097533	.115476	.016719	.090576	.035656	-.046231	.088309
Arginine---------------------do-----	-.000641	.002534	.000104	.002909	.002375	-.002557	.001615
Lysine-----------------------do-----	.010100	.018410	-.000811	.015028	.013707	-.010606	.012832
Methionine plus cystine-------do-----	.000240	-.000122	-.000430	-.000134	.000629	.000171	.000334
Methionine-------------------do-----	0	0	0	0	0	0	0
Tryptophan-------------------do-----	0	0	0	0	0	0	0
Calcium----------------------do-----	0	0	0	0	0	0	0
Inorganic phosphorus----------do-----	0	0	0	0	0	0	0
Whey factor------------------do-----	0	0	0	0	0	0	0
Fish factor------------------do-----	0	0	0	0	0	0	0
Change in formula cost---------dollar--	.314538	.151528	.060017	.109861	.183536	.319169	.133136

Ingredient or characteristic	Displacement rates and accompanying changes per ton of feed from increasing--			
	Tallow, 1.0 percent	Bone meal steamed, 1.0 percent	Phosphate dicalcium, 1.0 percent	Phosphate defluorinated, 1.0 percent
(1)	(28)	(29)	(30)	(31)
Ingredient:				
Soybean meal:				
Solvent, 44 percent------ton--	0.246913	-0.153719	-0.291586	-0.212582
Without hulls, 50 percent------do--	-.227113	.142484	.268627	.195922
Corn gluten meal, 41 percent------do--	.016122	.022605	.033801	.023325
Meat and bone scrap------do--	-.000815	-.026713	-.037206	-.026106
Cornmeal, yellow------do--	-.044832	.005477	.012811	.009460
Calcium, carbonate------do--	-.000356	.000026	.003855	.000197
Methionine------do--	.000080	-.000162	-.000302	-.000215
Nutritional characteristic:				
Productive energy per pound------calorie--	0	0	0	0
Protein------percent--	0	0	0	0
Nonfiber------do-----	-.769213	.433000	.860066	.629606
Arginine------do-----	-.021561	-.054820	-.067766	-.046849
Lysine------do-----	-.100480	-.009454	.017428	.015071
Methionine plus cystine------do-----	-.002642	.002342	.006217	.004428
Methionine------do-----	0	0	0	0
Tryptophan------do-----	0	0	0	0
Calcium------do-----	0	0	0	0
Inorganic phosphorus------do-----	0	0	0	0
Whey factor------do-----	0	0	0	0
Fish factor------do-----	0	0	0	0
Change in formula cost------dollar--	.119823	.802627	1.053394	1.016187

[1] See Appendix for information on the associated changes in vitamin content of formula.

When the divisions outlined above were made, the lowest cost means of increasing the amount of soybean meal without hulls appeared to be the addition of an inert filler. If a costless inert filler could be obtained, the increase could be brought about at an actual saving in cost. However, there is no truly costless filler and the margin of saving probably would not be large enough to cover the cost of a suitable filler. As the filler must be free of calcium and phosphorus, a pure silica sand would probably be the lowest cost filler in most locations. The saving in cost would not be large enough to cover the cost of pure silica sand in most areas. Moreover, there is resistance to the use of inert materials in feeds on the part of farmers, feed manufacturers, and feed regulatory authorities.[7] Thus, cost considerations alone probably would rule out the use of a filler but, even if it did not do so, other considerations would be likely to eliminate this possible source of increase in the amount of soybean meal without hulls.

The ingredient next in order of cost and probably lowest if cost of filler is taken into account was found to be wheat standard middlings, which is a satisfactory ingredient. The level of entry of this ingredient can be determined as follows. First determine the amount of soybean meal without hulls that is displaced by reducing the amount of dried corn distillers' grains with solubles, by 9.9610 percent. This is done by multiplying the indicated change in amount of soybean meal without hulls as given in column 17, table 4, by 9.9610. Next, divide the quantity displaced by the increase in amount of soybean meal without hulls per each percentage point increase in the amount of wheat standard middlings. This is given in column 22, as previously indicated. The quotient is the percentage level of wheat standard middlings in the formula from which dried corn distillers' grains, with solubles, are completely displaced. The arithmetic of this determination is:

$$(9.9610 \times 0.009793) \div 0.047436 = 2.0564$$

Thus, the least-cost adjustment of the initial formula for elimination of dried corn distillers' grains with solubles is obtained by multiplying the changes indicated in column 17 by -9.9610 and adding to these changes those obtained by multiplying 2.0564 by the changes indicated in column 22. These changes are summarized in table 5. The revised formula is given in table 6. Cost of the formula given in table 6 is $0.14 per ton more than that given in table 2.

If only partial displacement of dried corn distillers' grains with solubles had been desired, the percentage of desired displacement would have been used in the adjustment instead of 9.9610. The increase in cost would have been less by $0.14 ÷ 9.9610, or $.014 per percentage point of dried corn distillers' grains with solubles left in the formula.

It will be noted that soybean meal without hulls appears in neither the original formula given in table 2 nor in the revised formula of table 6. The concern with this ingredient in the above operation was one of achieving an "accounting" balance of the items that enter into the formula specification.

Comparison of the formulas given in tables 2 and 6 show that, in a sense, wheat standard middlings have taken the place of dried corn distillers' grains with solubles. But this is perhaps one of the least important changes. For example, corn gluten meal more than doubled in level of use between the formulas of tables 2 and 6. Also, the level of yellow cornmeal changed more in absolute amount than did the wheat standard middlings. This points up the fact that substitutions between two feed ingredients are rarely simple. Ordinarily, all other ingredients except those at minimum or required levels change in response to a change in level of any one ingredient.

The change in nutritional composition of the formula was confined to those items of specification that were above minimum or below maximum levels in the original formula of table 2 (see table 7). This is a general characteristic of changes of this type. Later in the report, changes in characteristics met exactly in the original formula are considered.

Some ingredients considered in this problem are not represented by columns in table 4. Displacement of any of these ingredients requires the use of one or more of the ingredients that are represented by

[7] The present instance does not make the case but presumably an occasion may arise in which a true saving in cost to the farmer could be realized by using inert filler in mixed feeds.

columns in table 4 which, if entered in the formula, reduce the ingredient in question.

Any one of 13 ingredients other than wheat standard middlings might have been used in making the adjustment. The change in cost of the formula if each of these other ingredients had been used, in turn, instead of wheat standard middlings would have been as follows:

Item	Cost	Item	Cost
	Dollars		*Dollars*
Fish solubles, condensed	.0387	Red dog flour	.4219
Dried-		Hominy feed, yellow	.0055
Whey, cheese	1.5395	Oats, feeding, rolled	.2004
Brewers' yeast	4.5341	Barley, excluding Pacific Coast	.0646
Alfalfa meal, dehydrated,		Bone meal, steamed	.2400
17 percent	.2440	Phosphate:	
Wheat:		Dicalcium	.0726
Hard red winter	1.8041	Defluorinated	.1973
Soft red winter	.7769		

From this list, it is obvious that condensed fish solubles, yellow hominy feed, barley, or several other ingredients might have been used instead of wheat standard middlings at a small additional sacrifice in cost. This is characteristic of many adjustments of this kind. No doubt, this is why broiler feed formulas vary between different areas, between mixers, and between periods of time. Small changes in cost or availability of ingredients result in substantial changes in least-cost formulas.

The reduction of synthetic methionine follows the same general procedure used above for reduction of dried corn distillers' grains with solubles. However, no column in table 4 relates to a change in synthetic methionine. An increase in any one of 18 ingredients that are represented by columns in table 4 will reduce the amount of synthetic methionine. Thus, the problem may be attacked indirectly by increasing the amounts of ingredients that reduce the level of synthetic methionine.

Increasing the Level of an Ingredient in the Formula

Purchaser preference, stocks of ingredients on hand, differences in minimum specifications of particular ingredients, and other causes may necessitate an increase in the level of an ingredient above the least-cost amount. For example, some mixers prefer to use a higher level of fish products than is called for in the minimum specifications for the feed formula. This may or may not be justified on nutritional grounds. Research evidence is not adequate enough to permit the setting of a firm minimum level of fish products.

This kind of adjustment parallels closely in logic that given above for the displacement of an ingredient. In the case of condensed fish solubles, column 11 of table 4 indicates the adjustment required in the formula to increase this ingredient by 1 percent. It may happen that an attempt to increase an ingredient calls for the displacement of an amount of some ingredient in excess of the amount in the formula. This does occur upon an increase in the amount of condensed fish solubles. This situation may be handled in the same way as was the problem of displacing dried corn distillers' grains with solubles, which was discussed previously.

When an increase is desired in the level of an ingredient that is not represented by a column in table 4, it is necessary to use one or more ingredients that are represented by columns in table 4 which require an increase in the ingredient in question.

ADJUSTING THE FORMULA TO MEET CHANGED SPECIFICATIONS

The nutritional requirements for boilers are probably as well established as are the requirements for any class of livestock. However, the problem of determining requirements is so complex that there is

TABLE 5.--SUMMARY OF CHANGES INVOLVED IN SUBSTITUTING WHEAT STANDARD MIDDLINGS FOR DRIED CORN DISTILLERS' GRAINS WITH SOLUBLES IN THE FORMULA PRESENTED IN TABLE 2

Item	-9.961 times values of col. 17, table 4	2.0564 times values of col. 22, table 4	Total of net changes for desired substitution
Soybean meal:			
Solvent, 44 percent------------------ton-----	0.094211	-0.110326	-0.016115
Without hulls, 50 percent------------do-----	-.097547	.097547	.000000
Corn gluten meal, 41 percent------------do-----	.069707	-.005439	.064268
Meat and bone scrap--------------------do-----	.001644	-.000228	.001416
Cornmeal, yellow-----------------------do-----	.032174	-.002367	.029807
Calcium, carbonate---------------------do-----	-.000468	.000278	-.000190
Methionine-----------------------------do-----	-.000110	-.000029	-.000139
Productive energy per pound----------calorie--	0	0	0
Protein------------------------------percent--	0	0	0
Nonfiber-------------------------------do-----	.276438	.237465	.513903
Arginine-------------------------------do-----	-.020370	.005211	-.015159
Lysine---------------------------------do-----	-.074658	.037858	-.036800
Methionine plus cystine----------------do-----	.007740	-.000251	.007489
Methionine-----------------------------do-----	0	0	0
Tryptophan-----------------------------do-----	0	0	0
Calcium--------------------------------do-----	0	0	0
Inorganic phosphorus-------------------do-----	0	0	0
Cost---------------------------------dollar--	-.180543	.311602	.131059

TABLE 6.--LEAST-COST FORMULA FOR GIVEN SPECIFICATIONS AND PRICES WHEN DRIED CORN DISTILLERS' GRAINS WITH SOLUBLES IS EXCLUDED

Ingredient	Amount	Percentage of formula	Cost
	Pound	*Percent*	*Dollars*
Soybean meal, solvent, 44 percent--------------------	313.75	15.688	9.44
Corn gluten meal, 41 percent------------------------	240.76	12.038	9.68
Meat and bone scrap---------------------------------	128.29	6.415	3.85
Fish solubles, condensed----------------------------	20.00	1.000	.92
Dried corn distillers' solubles---------------------	80.00	4.000	3.94
Alfalfa meal, dehydrated, 17 percent----------------	40.00	2.000	1.62
Cornmeal, yellow------------------------------------	1,106.61	55.330	32.31
Wheat standard middlings----------------------------	41.13	2.056	1.14
Calcium, carbonate----------------------------------	14.98	.749	.10
Salt--	5.00	.250	.04
Trace mineral supplement----------------------------	6.00	.300	.18
Methionine--	1.90	.095	5.00
Vitamins, 3-Nitro-4-hydroxyphenylarsonic acid, B.H.T. and penicillin[1]------------------------------------	1.58	.079	2.87
Total---	2,000.00	100.000	71.09

[1] For the amount of each item included in this group refer to the tabulation on page 5.

TABLE 7.--NUTRITIONAL COMPOSITION OF LEAST-COST FORMULA AT GIVEN PRICES WHEN DRIED CORN DISTILLERS' GRAINS WITH SOLUBLES ARE EXCLUDED COMPARED WITH SPECIFICATIONS

Characteristic	Level in-- Formula	Level in-- Specification	Amount in formula in excess of specification
Weight--------pound--	2000	2000	None
Productive energy per pound--------calorie--	950	950 or more	None
Protein--------percent--	22.6190	21.11 or more	1.51
Ratio of calories per pound to percentage of protein--------	42:1	42:1 and 45:1 range	
Nonfiber--------do-----	97.2	96 or more	1.2
Arginine--------do-----	1.2	1.2 or more	None
Lysine--------do-----	1.0	0.9 or more	0.1
Methionine--------do-----	0.48	0.48 or more	None
Methionine plus cystine--------do-----	0.8	0.8 or more	None
Tryptophan--------do-----	0.2	0.2 or more	None
Calcium--------do-----	1.1	1.1 exactly	None
Inorganic phosphorus--------do-----	.45	.45 exactly	None
Fish factor:			
Fishmeal, Menhaden or--------pound--	0	[1] 50	[2] None
Fish solubles, condensed--------do----	20	[1] 20	[2] None
Alfalfa factor:			
Alfalfa meal, dehydrated, 17 percent--------do----	40	40	None
Whey factor:			
Dried whey, cheese or--------do----	0	[1] 40	[3] None
Dried corn distillers' solubles, or--------do----	80	[1] 80	[3] None
Dried brewers' yeast--------do----	0	[1] 40	[3] None
Pigmenting factor:			
Corn gluten meal, 41 percent--------do----	112.22	60 or more	52.22
Vitamin:			
A--------million I.U.--	13.5	4.0 or more	9.5
D--------million I.C.U.--	1.5	.275 or more	1.225
K--------gram--	[4] 1.0	.432 or more	[4] .568
B_{12}--------milligram--	17.7	9.6 or more	8.1
Pantothenic acid--------gram--	12.2	10.0 or more	2.2
Niacin--------do---	45.3	28.8 or more	16.5
Choline--------do---	1524.0	1440.0 or more	84.0
Folic acid--------do---	1.5	.6 or more	.9
Riboflavin--------do---	6.5	3.1 or more	3.4

[1] Any single ingredient at level listed in specification or a linear combination of two or more of the ingredients will meet the "unknown growth factor" indicated.
[2] 20 pounds of fish solubles, condensed, meet the fish factor.
[3] 80 pounds dried corn distillers' solubles meet the whey factor.
[4] Possible vitamin content of feed ingredients is not included because of lack of reliable data on analysis.

room for differences of opinion concerning some of the specifications. The linear programming model is flexible enough to permit considerable departure from most of the specifications used in this problem.

Certain of the columns in table 4 do not relate to ingredients but rather to characteristics of the formula. Column 3, for instance, relates to the protein level. In fact, the first seven figure columns relate to changes in specifications.

Making the changes indicated in column 3 increases the amount of protein by 20 pounds. This operation is comparable to the previously described procedure for increasing the level of an ingredient. In this instance, in addition to the change in the percentage of protein in the formula, there are also changes in the characteristics that exceeded specifications in the original formula.

An increase in protein level alone lowers cost at the rate of $0.33 for each percentage point of protein added at the prices used in this problem. (Note that the "cost" entry of column 3, table 4, is $0.33+). However, a change of this kind, violates the requirement that the ratio of calories of productive energy to the percentage of protein be no closer than 42:1. If the amount of protein is to be increased and the ratio maintained, the amount of productive energy must be increased also. Column 4 indicates the changes that increase the caloric content of the formula by 50 calories per pound. Thus, to maintain a 42:1 ratio, this productive energy column must be entered at 42/50, or at an 84-percent level for each 1 percent of protein added. As the required increase in energy increases cost by 84 percent of $2.31, or $1.94, the total adjustment increases cost by $1.94 minus $0.33, or $1.61.

An increase of 0.1 percent in the allowance of inorganic phosphorus lowers the cost by $0.15+. (Note that the cost entry for column 8, table 4, is $0.15+). If this is done, it might be nutritionally desirable to increase the amount of calcium also in order to maintain a close approximation to the ratio of phosphorus to calcium that existed in the original specifications. An increase in amount of calcium increases cost, but the ratio of 45:1.1 between inorganic phosphorus and calcium could be maintained and the cost reduced about $0.07 per 0.1 percent increase in inorganic phosphorus. Opinions of nutritionists as to the advisability of making such a change may differ because of the existence of some degree of uncertainty concerning the precise levels and ratio of calcium and inorganic phosphorus that is optimum.

If an inert filler were added, this would be equivalent to lowering the weight requirement of the formula. Thus, this represents a change in specifications. If a costless filler could be obtained, this change would reduce cost, by $0.0227 per each percentage point of filler added. That is, up to $2.27 per ton could be paid for inert filler. As previously mentioned, fillers of this kind are not usually added. Moreover, obtaining a filler that is inert with respect to the formula specifications at such a low cost may not be possible.

All other increases in specifications represented by columns of table 4 result in cost increases.

There is no column in table 4 to indicate the changes that permit alteration of the original specifications for "nonfiber," "arginine," "lysine" and "methionine plus cystine." If a change in specifications for these elements is desired, it must be accomplished indirectly. That is, some ingredient or characteristic given in table 4 must be changed in such a way as to effect the desired change in the specification. The least-cost change is achieved by varying the level of the ingredient that increases the specification at lowest cost. The changes in specifications associated with change in the level of use of an ingredient (or other specification) are to be found in the nutritional characteristic section of table 4. Examination of the columns to determine the cost of increase parallels exactly the procedure outlined previously for finding the lowest cost source of increase in soybean meal without hulls.

ADJUSTING THE FORMULA FOR CHANGE IN INGREDIENT COMPOSITION

The composition of an ingredient is subject to variation from lot to lot. The linear programming model uses values for composition of ingredients given in table 1. Apparently, similar values are currently used by feed millers. If departure from the use of such values is desired, it is

relatively easy to make adjustments in the solution for changes in the composition of ingredients.

If the composition of an ingredient is below standard on one or more characteristics, the formula obtained by using the linear programming model will have less of the characteristic in which the ingredient is deficient than the solution indicates. To correct for this, the amount of the deficiency in units per ton of the ingredient is determined. Then, the appropriate adjustment for this level of deficiency is computed, using the relevant column of table 4, provided the specification is one that is met exactly by the formula. If the ingredient is not already in the formula, this adjustment is then added to the column in table 4 that relates to the ingredient. The adjusted ingredient column is then used in entering the ingredient into the solution.

If the characteristic that is deficient is not one that is met exactly by the solution, that is, if it is exceeded, the adjustment procedure differs somewhat. First, if the level of deficiency is small relative to the margin by which the formula exceeds specification for the characteristic in question, the deficiency may be ignored, as the formula would not be changed by taking such a deficiency into account. The formula would merely exceed the specifications by a narrower margin than is indicated, if the table value for the ingredient were used.

If the level of deficiency is large, then it is necessary to find, among the columns of table 4, the least-cost source of increase in the characteristic. The required level of adjustment is then computed in the same way as the adjustment outlined above, using the values given in this column.

If the ingredient is already in the mix, the correction involves determination of the appropriate adjustment by one of the methods outlined above. In this instance, one further refinement is necessary to prove that the adjusted solution is least-cost. In order to explain this refinement, it is necessary first to explain the development of the resulting net change in formula cost row of table 4. The entry in this row for a particular ingredient is developed by:

(1) Multiplying the price of each ingredient in the solution by the change in that ingredient,

(2) Summing the products obtained in (1),

(3) Subtracting the sum of products obtained in (2) from the cost of 1 percent of the ingredient for which the "resulting net change in formula cost" is being developed.

The price used in step (1) is for an ingredient having a composition as given in table I. If the ingredient is not of this composition, an adjusted "price" must be used. The adjustment consists of adding or subtracting the proportion of the resulting net change in formula cost for the specification that is required to correct for the ingredient's deficiency. The resulting adjusted price is then used instead of the true price in developing a new resulting net change in formula cost row in table 4. After the change, this new row may indicate that the cost of the formula would be reduced by adding an ingredient not previously in the solution.

If an ingredient exceeds the table values with respect to a characteristic, the reverse of the adjustment procedure outlined should be followed.

ADJUSTING THE SOLUTION FOR CHANGES IN PRICES

The discussion of the recomputation of the resulting net change in formula cost for table 4 in case of adjustment to meet departure of ingredients from their assumed characteristics can be extended easily to situations in which there is a true change in ingredient prices. A new cost can be obtained for any column by subtracting the value of ingredients increased in amount from the value of ingredients reduced. A negative difference indicates that adding the ingredient (or increasing the specification) lowers the cost of the formula.

The procedure for adding an ingredient to the formula was discussed previously. The same procedure is used in bringing in a cost-reducing ingredient.

EXTENDING CONSIDERATION TO INGREDIENTS NOT INCLUDED IN TABLE 4

Table 4 includes most of the ingredients commonly used in broiler foods. However, the list of possible ingredients is more extensive. Fortunately, it is relatively easy to add ingredients to the list given in table 4. Addition involves the use of the columns of table 4 that relate to changes in specifications. If 1 percent of an ingredient is added to the formula, the specifications for the residual of the formula can be lowered by the amount of each characteristic provided by the ingredient. Thus, a reduction of 1 percent of a ton of the residual ingredients is possible if 1 percent of an ingredient is added. To find the change in the formula involved in a reduction of 1 percent in weight, it is only necessary to refer to the values in column 2, table 4. If the ingredient has a protein percentage of 20, 1 percent of a ton contains 4 pounds of protein, or 0.2 percent of a ton of protein. If this ingredient is added to the formula at the 1-percent level, the specification for protein in the residual is reduced by 0.2 percent. If the signs are changed on items in column 3, table 4, the resulting changes meet the specifications for 1 percent less protein than was called for in the original specification for the formula. Therefore, minus one-fifth of the values given in column 3, table 4, gives the changes required to meet the specifications for a formula having 0.2 percent less protein than was called for in the original.

If the values of column 2 are added to the corresponding values of minus one-fifth, column 3, the resulting sums indicate the changes in the formula if an ingredient weighing 1 percent of a ton and containing 20 percent protein were to be added. In a comparable way, adjustment may be made for the energy, methionine, tryptophan, calcium, and inorganic phosphorus content of the ingredient to be added. This does not take into account the change in the level of characteristics in the formula that are not represented by columns in table 4. These are "nonfiber," "arginine," "lysine", and "methionine plus cystine." The full amount of the characteristic in the ingredient is to be added in developing the relevant row in table 4. Needless to say, the changes in the level of the characteristic resulting from changes previously outlined must be taken into account in arriving at the final entry.

The "resulting change in formula cost" entry for the column can be derived as outlined in the previous section, or it can be developed from the cost entries for the specifications columns and the price of the new ingredient.

A Linear Programming Model of the Broiler Feed Formula Problem

The model used in this problem evolved from several trial and error attempts at stating relevant relationships. Each trial after the first began with most of the numerical coefficients from a previous solution. This was an efficient procedure of model development but the final model used had some characteristics carried over from previous models that were not optimum for the final model. Therefore, instead of describing the linear programming model actually used in this problem, the model discussed here is the one that would be used if a new start were made on the problem with the knowledge gained from previous experience.[8]

As some of the specified characteristics of the formula could be met only by single ingredients, there was no point in introducing this characteristic into the model of the mix problem. Thus, salt, trace mineral supplement, corn gluten meal as a pigmenting factor, and alfalfa meal as a source of the alfalfa factor were added in fixed amounts to meet this type of characteristic. The whey and fish factors could be satisfied by more than one, but relatively few, ingredients. With some preliminary calculations, the choice could be narrowed to a single ingredient in each instance. Condensed fish solubles were chosen as the source of the fish factor and dried corn distillers' solubles were chosen as the source of the whey factor. This choice was made on the basis of prices used in obtaining the solution reported here. Changes in prices could make necessary a change in the sources of these factors to realize the least-cost formula. Procedures for making such substitutions are detailed in another section of this report.

As explained previously, an antioxidant, 3-Nitro-4-hydroxyphenylarsonic acid, an antibiotic, and vitamin supplements were added in fixed amounts. The insurance feature of adding these ingredients appears to be valued highly relative to the possible savings that might be realized by taking account of the amounts in the other ration ingredients.

[8] Initial and solution tableaus of the problem as run are available through the Computer Facility of Pennsylvania State University. These are available in either punched paper tape or tabular read out.

Table 8 gives the ingredients added in fixed amounts and their contributions toward satisfying the requirements of each characteristic of the formula.

The initial tableau is given in table 9. The column vectors, P_5, P_{47}, P_{48}, P_{49}, P_{50}, and P_{51}, may be omitted from the tableau without loss of value. They are included in table 9 for logical completeness. The formula specifications are represented by five columns in this tableau (table 9). These are: (1) P_0, which states the residual requirements of the formula for weight, fiber, productive energy, calcium, and inorganic phosphorus, as given in the tabulation on page 3 after account is taken of the fixed ingredients described in table 8; (2) P_{43}, which states the remaining residual requirements for a formula of exactly 950 calories, exactly 21.11 percent protein (45:1 ratio) and at least the residual levels of amino acids; (3) P_{45}, which states the remaining residual requirements for a formula of exactly 950 calories per pound, exactly 22.61 percent protein (42:1 ratio) and at least the residual levels of amino acids; (4) P_{44}, which states the possibility of increasing productive energy above the 950 calorie-per-pound level, provided the ratio of energy to protein is maintained at 45 calories per pound to 1 percent protein and that amino acids increase in direct ratio to productive energy and, finally; (5) P_{46}, which is the same as P_{44}, except that the energy-protein ratio is 42 to 1 instead of 45 to 1.

Either P_{43} or P_{45} must enter a solution to this problem. If P_{43} enters, then P_{44} may enter if the least-cost formula would have more than 950 calories per pound. Similarly, if P_{45} enters, then P_{46} may enter. If the optimum ratio of energy to protein is between the limits of 42:1 and 45:1, then either P_{43} and P_{46} or P_{45} and P_{44} would enter the solution.

Coefficients of a solution tableau were reported in table 4. This table did not include the coefficients of columns P_{43} through P_{51}, but all the information they convey can be derived from other columns.

TABLE 8.--INGREDIENTS ADDED TO FORMULA IN FIXED AMOUNTS IN ORDER TO SATISFY CERTAIN SPECIFICATIONS

Ingredient	Contribution to specifications					
	Weight	Protein	Fiber	Productive energy	Arginine	Lysine
	Pound	*Percent*	*Percent*	*Calories per pound*	*Percent*	*Percent*
Salt----------------------------	5.00	0	0	0	0	0
Mineral supplement--------------	6.00	0	0	0	0	0
Corn gluten meal----------------	60.00	1.287	.12	49.26	.042	.024
Dried corn, distillers' solubles	80.00	1.076	.152	81.60	.040	.036
Fish solubles, condensed--------	20.00	.314	.006	8.80	.024	.027
Alfalfa meal, dehydrated, 17 percent-----------------------	40.00	.356	.484	10.44	.016	.018
Vitamin, 3-Nitro-4-hydroxy-phenylarsonic acid, B.H.T. & penicillin[1]-----------------	1.58	0	0	0	0	0
Total----------------------	212.58	3.033	.762	150.10	.122	.105

Ingredient	Methionine	Methionine plus cystine	Tryptophan	Calcium	Inorganic phosphorus
	Percent	*Percent*	*Percent*	*Percent*	*Percent*
Salt----------------------------	0	0	0	0	0
Mineral supplement--------------	0	0	0	0	0
Corn gluten meal----------------	.030	.048	.006	.005	.0035
Dried corn distillers' solubles-	.024	.048	.008	.014	.016
Fish solubles, condensed--------	.010	.027	.008	.006	.007
Alfalfa meal, dehydrated, 17 percent---------------------	.0065	.013	.0045	.034	.001
Vitamin, 3-Nitro-4-hydroxy-phenylarsonic acid, B.H.T. & penicillin[1]-----------------	0	0	0	0	0
Total----------------------	.0705	.136	.0265	.059	.0275

[1] For the amount of each item included in this group refer to the tabulation on page 5.

Illustration of a Change in Formula to Meet a Change in Specifications

There is a paucity of experimental data to explain the relationship of ration efficiency to the energy-protein ratio. In general, it is known that a feed that includes protein in excess of the biological requirements for growth is less efficient per calorie in producing gain than is a ration that does not contain an excess of protein. The precise biological requirement of protein for growth at the various ages has not yet been established; but existing protein standards are known to give tolerable results. However, as the consequences of too little protein are more serious than those of mild excesses of protein, the standards may be somewhat above the true minima.

As a result of this uncertainty as to precise requirements, some nutritionists may wish to depart from the energy-protein ratio range given in the specifications for this formula. For instance, a narrower range may be favored for a feed during the first 2 to 3 weeks of growth. With no position taken on the advisability of one protein

TABLE 9.--INITIAL TABLEAU FOR LEAST-COST BROILER FEED FORMULA[1]

$C_j \rightarrow$			-M	-M		-M	-M						-M	-M	-60.20	-67.20	-80.40
$\downarrow$		P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
-M	P_1	1787.42	+1												2000	2000	2000
-M	P_2			+1											45.8	50.9	42.9
	P_3	3.24			+1										5.8	2.8	4.0
-M	P_4					+1									1,520	1,580	1,642
-M	P_5	1749.9					+1								1,520	1,580	1,642
-M	P_{47}							-1							3.20	3.58	1.4
-M	P_{48}								-1						2.90	3.58	.8
-M	P_{49}									-1					.62	.75	1.0
-M	P_{50}										-1				1.28	1.49	1.6
-M	P_{51}											-1			.60	.65	.2
-M	P_{11}	1.04											+1		.32	.26	.16
-M	P_{12}	.42												+1	.20	.19	.12

$C_j \rightarrow$		-76.60	-1060.00	-1150.60	-1092.10	-1325.20	-1225.20	-1123.00	-210.00	-98.50	-63.20	-80.80
$\downarrow$		P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}
-M	P_1	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
-M	P_2	53.4	50.6	61.3	31.4	32.0	33.5	13.1	44.6	26.9	27.2	17.8
	P_3	2.4	2.2	.7	.6	.4	.2	.3	2.7	3.8	9.0	24.2
-M	P_4	1,898	1,748	1,882	880	1,572	1,530	1,572	1,144	2,040	1,782	522
-M	P_5	1,898	1,748	1,882	880	1,572	1,530	1,572	1,144	2,040	1,782	522
-M	P_{47}	3.7	4.0	4.0	2.4	1.1	1.2	.2	2.2	1.0	.9	.8
-M	P_{48}	3.8	3.5	5.3	2.7	2.4	2.8	.8	3.0	.9	.7	.9
-M	P_{49}	.8	.7	1.80	1.0	.7	.8	.15	.7	.6	.5	.32
-M	P_{50}	1.4	1.3	2.8	2.7	1.0	1.3	.46	1.2	1.2	.8	.66
-M	P_{51}	.3	.2	.6	.8	.5	.4	.1	.5	.2	.1	.23
-M	P_{11}	7.94	10.57	5.49	.61	1.34	1.26	.9	.13	.35	.17	1.7
-M	P_{12}	4.03	5.07	2.81	.70	.94	1.03	.8	.43	.40	.20	.06

See footnote at end of table.

-Continued

TABLE 9.--INITIAL TABLEAU FOR LEAST-COST BROILER FEED FORMULA[1]--Continued

$C_j \rightarrow$		-58.40	-65.00	-1000.	-79.80	-55.20	-1055.90	-54.20	-62.20	-97.60	-56.20	-1147.20	-1004.00
$\downarrow$		P_{27}	P_{28}	P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}
-M	P_1	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
-M	P_2	8.9	11.3	15.2	10.2	17.2	.16	11.1	12.0	15.0	12.7		12.1
	P_3	2.0	2.2	2.6	2.1	7.6	.3	4.9	11.0	2.0	5.4		1.7
-M	P_4	2,210	2,198	1,794	1,794	1,368	2,000	1,670	1,620	2,324	1,626	5,756	600
-M	P_5	2,210	2,198	1,794	1,794	1,368	2,000	1,670	1,620	2,324	1,626	5,756	600
-M	P_{47}	.4	.3	.5	.3	.9	1.0	.5	.6	.96	.5		
-M	P_{48}	.3	.3	.4	.3	.7	.6	.4	.4	.61	.3		
-M	P_{49}	.14	.16	.21	.14	.2	.1	.1	.13	.24	.12		
-M	P_{50}	.29	.36	.45	.3	.4	.3	.23	.35	.51	.32		
-M	P_{51}	.08	.09	.16	.11	.2	.2	.1	.14	.21	.13		
-M	P_{11}	.02	.03	.05	.04	.15	.10	.05	.09	.07	.09		28.98
-M	P_{12}	.09	.09	.12	.09	.27	.15	.16	.12	.12	.12		13.59

$C_j \rightarrow$		-.68	-10004.295	-52.67	-4.580					-M	-M	-M	-M	-M
$\downarrow$		P_{39}	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	P_{50}	P_{51}
-M	P_1	2000	2000	2000	2000									
-M	P_2					-18.08	-21.11	-19.59	-22.62					
	P_3													
-M	P_4					-1749.9		-1749.9						
-M	P_5						-1900		-1900					
-M	P_{47}					-1.08	-1.20	-1.08	-1.20	+1				
-M	P_{48}					-.80	-.90	-.80	-.90		+1			
-M	P_{49}			100.0		-.41	-.48	-.41	-.48			+1		
-M	P_{50}			100.0		-.66	-.80	-.66	-.80				+1	
-M	P_{51}					-.17	-.20	-.17	-.20					+1
-M	P_{11}	36.59	27.00		28.19									
-M	P_{12}		19.07		13.40									

Key to processes:

P_1 Weight disposal.

P_2 Protein disposal.

P_3 Fiber disposal.

P_4 Productive energy-protein ratio disposal.

P_5 Productive energy disposal.

P_6 Arginine disposal.

P_7 Lysine disposal.

P_8 Methionine disposal.

P_9 Methionine plus cystine disposal.

P_{10} Tryptophan disposal.

P_{11} Calcium disposal.

P_{12} Inorganic phosphorus disposal.

P_{13} Soybean meal, solvent, 44 percent.

P_{14} Soybean meal, without hulls, 50 percent.

P_{15} Corn gluten meal, 41 percent.

P_{16} Meat scrap.

P_{17} Meat and bone scrap.

P_{18} Fishmeal, Menhaden.

P_{19} Fish solubles, condensed.

P_{20} Dried buttermilk.

P_{21} Dried skim milk.

P_{22} Dried whey, cheese.

P_{23} Dried brewers' yeast.

P_{24} Dried corn distillers' solubles.

P_{25} Dried corn distillers' grains, with solubles.

P_{26} Alfalfa meal, dehydrated, 17 percent.

P_{27} Cornmeal, yellow.

P_{28} Milo, maize.

P_{29} Wheat, hard red winter.

P_{30} Wheat, soft red winter.

P_{31} Wheat, standard middlings.

P_{32} Wheat, red dog flour.

P_{33} Hominy feed, yellow.

P_{34} Oats, excluding Pacific Coast.

P_{35} Oats, feeding, rolled.

P_{36} Barley, excluding Pacific Coast.

P_{37} Tallow.

P_{38} Bonemeal, steamed.

P_{39} Calcium, carbonate.

P_{40} Dicalcium phosphate.

P_{41} Methionine supplement.

P_{42} Defluorinated phosphate.

P_{43} Mix with ratio of productive energy to protein of 45 calories per pound to 1 percent protein and 950 calories per pound.

P_{44} Process permitting increase in calories above 950 per pound and requiring that total protein be increased in the ratio of 1 percent protein to 45 calories per pound and that the lower limit of amino acids remain in the same ratio to energy as in the formula specifications.

P_{45} Mix with ratio of productive energy to protein of 42 calories per pound to 1 percent protein and 950 calories per pound.

P_{46} Same as P_{44} except that the ratio of productive energy to protein is 1 percent to 42 calories.

P_{47} Artificial source of arginine.

P_{48} Artificial source of lysine.

P_{49} Artificial source of methionine.

P_{50} Artificial source of methionine plus cystine.

P_{51} Artificial source of tryptophan.

level or another, the techniques for moving from one ratio to another are demonstrated.

It is assumed that a feed of the same specifications as given in table 3 is desired, except that the protein level will be exactly 24 instead of 22.6 percent--the maximum for a 950-calorie feed and a range of 42:1 to 45:1 in the energy-protein ratio. The energy-protein ratio in a 950-calorie formula having 24 percent protein is about 39:1. An increase of 1.4 percent in protein level is required to reach 24 percent.

The changes in ingredients, specifications, and cost are obtained by multiplying column 3, table 4, by 1.4. For clarity of presentation, this column from table 4 is reproduced in table 10, together with 1.4 times the values indicated.

Reference to the solution given in table 2 shows that the changes indicated above are feasible. However, these changes alone will not necessarily yield the least-cost 24-percent formula. The entry of soybean meal without hulls may make possible a cost-reducing adjustment in the formula. To test for this possibility, the columns of table 4 are examined to determine whether (1) increase in any ingredient (or specification) that reduces cost also reduces the amount needed of soybean meal without hulls, or (2) reduction of any ingredient (or specification) that will lower cost also reduces the amount needed of soybean meal without hulls. Normally, the search outlined under (1) is unnecessary except after a price change. Only a few ingredients in the formula are represented also by columns in table 4. Thus, the search outlined under (2) is relatively simple.

The ingredients that are represented by columns in table 4 and that are also in the formula given in table 2 are: Fish solubles, condensed; dried corn distillers' solubles; dried corn distillers' grains, with solubles; alfalfa meal, dehydrated, 17 percent.

Reducing the amount of dried corn distillers' grains with solubles will reduce cost and also the amount needed of soybean meal without hulls. Reducing the amount of condensed fish solubles would reduce both cost and the amount of soybean meal without hulls, but this would leave the fish factor unsatisfied. Table 4 indicates that condensed fish solubles are the cheapest source of the fish factor; therefore, a reduction in the amount of this ingredient would not be a saving. Reducing the amount of dried corn distillers' solubles increases the amount of soybean meal without hulls. Thus, its reduction was not limited by there being none of this ingredient to displace. It will remain the lowest-cost source of the whey factor.

The only apparently feasible change, therefore, is a reduction in the amount of dried corn distillers' grains with solubles. The changes associated with a reduction in the amount of this ingredient can be derived from column 17, table 4. Each percentage point reduction will reduce the amount of soybean meal without hulls by 0.009793 tons. As the proposed change to 24-percent protein would increase the amount of soybean meal without hulls by 0.141992 tons, there is more than enough to permit complete displacement of the 9.9610 percent of dried corn distillers' grains with solubles from the formula. The change involved can be determined by multiplying the changes indicated in column 17, table 4, by -9.9610. The result of this operation is summarized in table 10.

The total change in the formula to obtain the least-cost 24-percent protein formula is obtained by summing the changes indicated for an increase of 1.4 percent and those that indicate a reduction of 9.9610 percent in dried corn distillers' grains with solubles. The result is shown in the final column of table 10.

The resulting formula is shown in table 11.

The cost of this formula at prices used in the illustration is $70.30 per ton. This is $0.65 lower than the formula held to a maximum of 22.6 percent protein (that is, 42:1 ratio). If this feed is as efficient as the wider ratio feeds, a substantial cost advantage would be realized.

Stability of the Least-Cost Solution to Price Changes

As was seen in the illustration of the displacement of dried corn distillers' grains with solubles, there may be rather drastic

TABLE 10.--LEAST-COST FORMULA WITH GIVEN PRICES, WITH 24 PERCENT PROTEIN AND WITH OTHER SPECIFICATIONS UNCHANGED

Characteristic	Values from table 4		Changes indicated to-		
	Column 2 (protein)	Column 16 (dried corn distillers' grains with solubles)	Increase protein 1.4 percent col. (2) x 1.4	Reduce dried corn distillers' grains with solubles, 9.9610 percent col. (3) x -9.9610	Obtain least-cost 24-percent protein ration col. (4) + col. (5)
(1)	(2)	(3)	(4)	(5)	(6)
Soybean meal:					
Solvent, 44 percent---------------ton--	-0.118808	-0.009458	-0.166331	.094211	-.072120
Without hulls, 50 percent----------do--	.101423	.009793	.141992	-.097548	.044444
Corn gluten meal, 41 percent---------do--	.032525	-.006998	.045535	.069707	.115242
Meat and bone scrap-----------------do--	.000387	-.000165	.000542	.001644	.002186
Cornmeal, yellow--------------------do--	-.015268	-.003230	-.021375	.032174	.010799
Calcium, carbonate------------------do--	.000073	.000047	.000101	-.000468	-.000366
Methionine--------------------------do--	-.000331	.000011	-.000463	-.000110	-.000573
Productive energy per pound-----calorie--	0	0	0	0	0
Protein-----------------------percent--	1.000000	0	1.400000	0	1.400000
Nonfiber-------------------------do-----	.304687	-.027752	.426562	.276438	.703000
Arginine-------------------------do-----	.023885	.002045	.033439	-.020370	-.013069
Lysine---------------------------do-----	.041345	.007495	.057883	-.074658	-.016775
Methionine plus cystine----------do-----	.014061	-.000777	.019685	.007740	.027425
Methionine-----------------------do-----	0	0	0	0	0
Tryptophan-----------------------do-----	0	0	0	0	0
Calcium--------------------------do-----	0	0	0	0	0
Inorganic phosphorus-------------do-----	0	0	0	0	0
Cost---------------------------dollar--	-.332488	.018125	-.465483	-.180543	-.646026

TABLE 11.--LEAST-COST FORMULA WITH GIVEN PRICES, WITH 24 PERCENT PROTEIN AND WITH OTHER SPECIFICATIONS UNCHANGED

Ingredient	Amount		
	Quantity	Percentage	Cost
Soybean meal:	*Pounds*	*Percent*	*Dollars*
Solvent, 44 percent	201.74	10.087	6.07
Without hulls, 50 percent	88.89	4.444	2.99
Corn gluten meal, 41 percent	342.70	17.135	13.77
Meat and bone scrap	129.83	6.492	3.89
Fish solubles, condensed	20.00	1.000	.92
Dried corn distillers' solubles	80.00	4.000	3.94
Alfalfa meal, dehydrated, 17 percent	40.00	2.000	1.62
Cornmeal, yellow	1068.60	53.430	31.20
Calcium, carbonate	14.63	.731	.10
Salt	5.00	.250	.04
Trace mineral supplement	6.00	.300	.18
Methionine	1.03	.052	2.71
Vitamins, 3-Nitro-4-hydroxyphenylarsonic acid, B.H.T. and penicillin[1]	1.58	.079	2.87
Total	2000.00	100.000	70.30

[1] For the amount of each item included in this group see the tabulation on page 5.

The computed analysis of the formula in table 11 is:

Characteristics	Amount
Protein--percent	24.0
Nonfiber--do	96.8
Productive energy per pound--calorie	950.0
Arginine--percent	1.2
Lysine--do	1.1
Methionine--do	.48
Methionine plus cystine--do	.8
Tryptophan--do	.20
Calcium--do	1.10
Inorganic phosphorus--do	.45
Vitamin:	
A--million I.U.	14.6433
D--million I.C.U.	1.5000
K--gram	1.0000
B_{12}--milligram	17.8088
Pantothenic acid--gram	12.1019
Niacin--do	45.1324
Choline--do	1484.9903
Folic acid--do	1.5153
Riboflavin--do	6.5009

changes in the formula with small change in cost. From this, it may be deduced that small changes in the prices of ingredients may result in a considerable change in the least-cost formula. This is true for prices of some ingredients but drastic changes in prices of other ingredients are necessary before the formula is changed materially.

The sensitivity of the formula to prices of ingredients is confined to changes in relative price alone. That is, if all prices rose or fell by the same percentage, the formula would not change.

An infinitely large number of changes in relative price of the 30 ingredients considered is possible. Thus, it is obviously impossible to try to account for changes in the formula in response to all possible sets of relative prices. However, the extent of change in the price of each ingredient that is required before the least-cost formula changes, on the assumption that all other prices remain the same, indicates something of the degree of stability in the formula developed from the given prices. The percentage changes required, computed as indicated, are shown in table 12.

TABLE 12.--PERCENTAGE CHANGE IN PRICE REQUIRED TO REDUCE OR INCREASE LEVEL OF INGREDIENTS IN LEAST-COST FORMULA

Ingredient	Change in price to--	
	Reduce level	Increase level
	Percent	*Percent*
Soybean meal:		
Solvent, 44 percent	0.1	-0.3
Without hulls, 50 percent	---	-.1
Corn gluten meal, 41 percent	.1	-30.2
Meat scrap	---	-8.9
Meat and bone scrap	9.1	-13.7
Fish-		
Meal, Menhaden	---	-18.4
Solubles, condensed	---	-22.3
Dried-		
Buttermilk	---	-79.0
Skim milk	---	-68.3
Whey, cheese	---	-65.7
Brewers' yeast	---	-73.9
Corn distillers'-		
Solubles	---	-25.7
Grains, with solubles	---	-2.9
Alfalfa meal, dehydrated, 17 percent	---	-65.7
Cornmeal, yellow	4.6	-.4
Milo, maize	---	-10.4
Wheat:		
Hard red winter	---	-46.4
Soft red winter	---	-39.4
Standard middlings	---	-27.5
Red dog flour	---	-10.7
Hominy feed, yellow	---	-20.3
Oats:		
Excluding Pacific Coast	---	-29.5
Feeding, rolled	---	-32.7
Barley, excluding Pacific Coast	---	-23.7
Tallow	---	-8.1
Bonemeal, steamed	---	-100.3
Calcium, carbonate	474.9	-10.4
Phosphate:		
Dicalcium	---	-122.5
Defluorinated	---	-100.9
Methionine	1.8	-.3

Procedure for Economizing on Vitamin Supplement

The same fixed vitamin supplement, that given in the tabulation on page 5, was used with all the formulas presented here. In each formula, the vitamin content exceeded by a considerable margin the allowances specified in the tabulation on page 3. The margin of excess varied between the several formulas. In all of them, however, there existed the possibility of reducing some of the vitamin supplements and thereby reducing the cost of the formula.

As mentioned previously, economizing on vitamins to a point at which there is a chance of a deficiency occurring can be serious. This would argue for a comfortable margin of safety in the vitamin content. Feed manufacturers seem generally to hold this viewpoint. But, a more systematic approach to the problem would appear to be that of increasing the allowances for vitamins to provide the desired margin of safety. Retaining an inflated set of allowances may give better insurance against deficiency and be less costly than adding fixed amounts of supplement and realizing a variable margin of safety.

A procedure is outlined in this section for adjusting the level of vitamin supplement to a prescribed minimum level. The minimum level prescribed is made up of the allowances given in the tabulation of specifications on page 3. However, the procedure is the same for adjusting to any other level.

If the amount of vitamin supplement is varied to maintain the vitamin content at a prescribed minimum level, the relationship between the relative value of the ingredients may be changed. If the adding of one ingredient, plus its associated changes, results in the need for either more or less vitamin supplement than the adding of another ingredient, plus its associated changes, this should be taken into account. A procedure for accomplishing this is also outlined here.

Adjusting the level of vitamin supplement to a prescribed level.--Table 3 gives the amount by which the formula given in table 2 exceeds specification for each vitamin. As a first approximation, the vitamin supplement may be reduced until either all the supplement is displaced or the content of the formula is reduced to specification. The amount of each vitamin supplement added is given in the tabulation on page 5. The following tabulation was developed by taking the smaller of the two limits described above upon removal of supplements. The vitamins that may be removed weigh 342 grams, or 0.75 pound. The removal of this weight will require a

Vitamin	Amount of supplement that may be removed (1st approximation)
A----------million I.U.--	4.000
D----------million I.C.U.--	1.225
K----------gram--	.568
B_{12}----------milligram--	6.000
Panthothenic acid----------gram--	2.300
Niacin----------do---	17.800
Choline----------do---	293.800
Folic acid----------do---	1.000
Riboflavin----------do---	4.000

corresponding increase in the weight of other ingredients. This can be accomplished by using column 2 of table 4. The vitamin supplements removed can be considered inert filler as they have no nutritive value other than their vitamin content. However, the change resulting from an increase in the weight of other ingredients will itself alter the vitamin content of the mix. For each percentage point of inert filler added, the changes in vitamin content of the formula are as follows:

Vitamin	Change in vitamin content of formula per 1 percent increase in inert filler
A------million I.U.--	-0.186574
B_{12}------milligram--	.072486
Pantothenic acid------gram--	-.185233
Niacin------do---	-1.327252
Choline------do---	-23.442758
Folic acid------do---	-.004898
Riboflavin------do---	-.065815

The values for vitamins D and K are not given as none of the ingredients except the supplement contain vitamin D, and reliable estimates of the K content are not available for all ingredients. No contribution from ingredients of vitamin K was assumed in calculating the reduction permitted in the supplements.

The change in vitamin content of the formula caused by the reduction of 0.75 pound in inert filler can be obtained by multiplying -(.75 ÷ 20), or -.0375, by the values given in the tabulation above. The tabulation that follows gives the products of this multiplication.

Vitamin	Change in vitamin content caused by reducing inert filler 0.75 pounds
A------million I.U.--	0.006997
B_{12}------milligram--	-.002718
Pantothenic acid------gram--	.006946
Niacin------do---	.049772
Choline------do---	.879103
Folic acid------do---	.000184
Riboflavin------do---	.002468

The increase in vitamins A, folic acid, and riboflavin will not permit more of the vitamin supplements to be displaced as the previous first approximation of the amount that could be displaced was limited by the amount of these supplements added. Less of B_{12} and more of pantothenic acid, niacin, and choline may be displaced than was indicated by the first approximation. The net change in weight of vitamin supplement caused by the reduction of 0.75 pound of inert filler is less than one gram, or 0.002 pound. This change in weight would have an imperceptible effect on the formula; therefore, a final approximation of the change in vitamin supplement can be developed by adjusting the first approximation for the change in B_{12}, pantothenic acid, niacin, and choline by the amounts resulting from reducing inert filler by 0.75 pound. This final approximation is given in the tabulation below. Digits are carried beyond the limit of significance in this tabulation to better illustrate the nature of the calculation. Very little accuracy would have been sacrificed by accepting the first approximation as the final adjustment.

Vitamin	Amount of supplement that may be removed (final adjustment)
A-------million I.U.--	4.000
D-------million I.C.U.--	1.225
K-------gram--	.568
B_{12}-------milligram--	5.997
Pantothenic acid-------gram--	2.307
Niacin-------do---	17.850
Choline-------do---	294.679
Folic acid-------do---	1.000
Riboflavin-------do---	4.000

The change in formula cost that is due to the change outlined in vitamin supplements can be calculated by applying the prices for vitamin supplements as given in the tabulation on page 10 to the amounts of reduction as given in the tabulation above and by computing the change in cost caused by the reduction in amount of inert filler. The computed saving on vitamin supplements is $1.52. (More than a third of this amount is due to the reduction in vitamin A supplement.) The increase in cost that comes from the reduction in the amount of inert filler is less than one mill.

Adjusting the relative values of ingredients for vitamin value.--For each of the changes represented by columns in table 4, the associated net change in vitamin composition of the formula was computed. These values are given in table 13. Attention centers on the changes associated with the changes in ingredients discussed here. The procedures outlined are applicable also to changes in specifications.

The adjustment required in the relative value of the ingredients, as reflected in the net change in cost row of table 4, depends upon which vitamins would fall below specification without the added supplements and which vitamins would be subject to economizing. If all vitamins fell below specification without supplement and all were subject to economizing the relevant adjustment would be developed by multiplying the price of each vitamin supplement by the change in vitamin content and summing the products obtained. If the sum of products is negative, this indicates that the value of the supplements that need to be increased in amount exceeds the value of supplements that may be reduced in amount. Conversely, if the sum of products is positive, the value of the supplements that may be reduced exceeds the value of supplements that must be increased. These sums of products should be adjusted for the change in weight associated with the change in vitamin supplements. This adjustment is so small, however, that it can be safely ignored.

To demonstrate the change in relative value of ingredients if all vitamin supplements are to be economized, table 14 was prepared. The resulting net change in formula cost is taken from table 4. Net cost of vitamin supplements was computed from the change in vitamin content given in table 13 and the prices for supplements given in the tabulation on page 10. The net change in formula cost with no change in vitamin content was computed from the other two columns. For the most part, the changes are relatively small. Condensed fish solubles, and dehydrated alfalfa meal in particular are relatively better buys and tallow a relatively poorer buy if vitamins are treated in this way.

It should be noted, however, that the method used in computing the vitamin values for ingredients in table 14 will seldom be relevant. First, not all vitamin supplements will be economized. For example, it is doubtful whether a manufacturer would ever feel safe in mixing a formula without any vitamin A supplement. Second, the least-cost mix (not economizing on vitamin supplements) will sometimes have vitamin content from ingredients in excess of specifications. For example, the least-cost formula reported in table 2 would have had an excess over specifications of vitamins A, B_{12}, and riboflavin and would have met

TABLE 13.--CHANGE IN VITAMIN CONTENT OF FORMULA RESULTING FROM CHANGE IN INGREDIENTS OR SPECIFICATIONS

Ingredient or specification	Increase in ingredient or specification	Associated change in vitamins-						
		A	B_{12}	Pantothenic acid	Niacin	Choline	Folic acid	Riboflavin
		Million I.U.	*Milligrams*	*Grams*	*Grams*	*Grams*	*Grams*	*Grams*
Inert filler------------------------	1.0 percent	-0.186574	0.072486	-0.185233	-01.327252	-023.442758	-0.004898	-0.065815
Protein-----------------------------	1.0 do.	.736188	.034675	-.001825	.435834	-37.316540	-.005364	-.040625
Productive energy-------------------	50 calories per lb.	-.673272	.252762	-.630195	-5.503147	-94.804008	-.015049	-.252309
Methionine--------------------------	.1 percent	-.018645	.007258	-.018580	-.132877	-2.355822	-.000492	-.006597
Tryptophan--------------------------	.1 do.	-6.812760	-.300787	.762155	-7.378195	535.920010	.078406	.333100
Calcium-----------------------------	.1 do.	-.051002	.019802	-.050762	-.362940	-6.432376	-.001344	-.018016
Inorganic phosphorus----------------	.1 do.	-.492966	1.758131	-.001796	.765074	55.282848	.029813	.101746
Meat scrap--------------------------	1.0 do.	-.006873	-.245813	-.001473	.211525	-.933024	.002343	.015497
Fishmeal, Menhaden------------------	1.0 do.	.067494	.211533	.002004	.399339	6.711520	.010648	.012970
Fish solubles, condensed------------	1.0 do.	.246026	5.930445	.136427	1.125445	-12.968806	-.008537	.069025
Dried buttermilk--------------------	1.0 do.	.078796	.034288	.158105	-.169566	-11.213734	-.003743	.245398
Dried skim milk---------------------	1.0 do.	.001010	.230099	.194904	-.255447	-9.717602	-.001216	.147286
Dried whey, cheese------------------	1.0 do.	-.064755	.040717	.358960	-.370911	5.371562	.003111	.131312
Dried brewers' yeast----------------	1.0 do.	-.074098	-.033242	.849128	3.525061	10.351506	.082046	.278725
Dried corn, distillers' solubles----	1.0 do.	-.078375	-.053850	.121058	.862919	37.081014	.007111	.140770
Dried corn, distillers' grains, with solubles------------------------	1.0 do.	-.130063	-.014784	.021284	.213289	19.733088	.005291	.063359
Alfalfa meal, dehydrated, 17 percent	1.0 do.	1.886472	.045875	.085238	-.713197	-15.427958	.073686	.095489
Milo maize--------------------------	1.0 do.	-.055572	-.000269	-.045389	.156933	1.256336	.000034	-.006631
Wheat, hard red winter--------------	1.0 do.	-.062137	.005376	.042083	.188351	-1.010176	.001003	-.010808
Wheat, soft red winter--------------	1.0 do.	-.051180	-.024192	.045672	.159002	-1.156158	.000616	-.012962
Wheat, standard middlings-----------	1.0 do.	-.070396	-.009946	.066750	.380736	-6.453086	.003921	-.015429
Wheat, red dog flour----------------	1.0 do.	-.025371	-.004211	.050101	.321822	-.299830	.005154	-.002903
Hominy feed, yellow-----------------	1.0 do.	.023612	.001165	-.009424	-.000782	-1.047788	-.000058	-.002083
Oats, excluding Pacific Coast-------	1.0 do.	-.063858	.010214	.042640	-.225828	-2.775994	-.001275	-.017812
Oats, feeding, rolled---------------	1.0 do.	.009254	.006541	.077844	.126938	2.382154	.002802	.002877
Barley, excluding Pacific Coast-----	1.0 do.	-.075433	.009587	-.018224	.084933	-.923518	.002252	-.009050
Tallow------------------------------	1.0 do.	.200953	-.073024	.177009	1.839443	31.032602	.003744	.079301
Bonemeal, steamed-------------------	1.0 do.	.582442	-2.393485	.023841	-1.000557	-69.820064	-.039986	-.123930
Phosphate dicalcium-----------------	1.0 do.	.891253	-3.333658	-.045213	-1.086799	-111.588130	-.058140	-.211294
Phosphate defluorinated-------------	1.0 do.	.617752	-2.339098	-.040225	-1.330159	-79.480304	-.041076	-.151473

TABLE 14.--ILLUSTRATIVE PROBLEM: CHANGE IN FORMULA COST AS GIVEN IN TABLE 4 AFTER ADJUSTMENT FOR NET VALUE OF VITAMIN CHANGE, RESULTING FROM ALTERNATIVE CHANGES IN INGREDIENTS AND SPECIFICATIONS

Ingredient or specification	Change in ingredient or specification	Changes in formula cost (from table 4)	Value of change in vitamins at given prices	Change in formula cost adjusted for change in value of vitamins
		(1)	(2)	col. (1) - col. (2)
		Dollars	*Dollars*	*Dollars*
Inert filler	1.0 percent	-0.022744	-0.065636	0.042892
Protein	Increase 1.0 do.	-.332488	.070468	-.402956
Productive energy	Increase 50 calories per pound	2.313428	-.251788	2.565216
Methionine	Increase 0.1 percent	5.264278	-.006576	5.270854
Tryptophan	Increase 0.1 do.	.914302	-.476340	1.390642
Calcium	Increase 0.1 do.	.030837	-.017972	.048809
Inorganic phosphorus	Increase 0.1 do.	-.158721	.047231	-.205952
Meat scrap	Increase 1.0 do.	.068085	-.006801	.074886
Fishmeal, Menhaden	Increase 1.0 do.	.277007	.026266	.250741
Fish solubles, condensed	Increase 1.0 do.	.205693	.216386	-.010693
Dried buttermilk	Increase 1.0 do.	2.569625	.016153	2.553472
Dried skim milk	Increase 1.0 do.	1.538350	.008695	1.529655
Dried whey, cheese	Increase 1.0 do.	.808085	.011727	.796358
Dried brewers' yeast	Increase 1.0 do.	1.551352	.068736	1.482616
Dried corn distillers' solubles	Increase 1.0 do.	.253001	.039593	.213408
Dried corn distillers' grains, with solubles	Increase 1.0 do.	.018125	.005294	.012831
Alfalfa meal dehydrated, 17 percent	Increase 1.0 do.	.530476	.254510	.275966
Milo maize	Increase 1.0 do.	.067404	-.007102	.074506
Wheat, hard red winter	Increase 1.0 do.	.440329	-.007092	.447421
Wheat, soft red winter	Increase 1.0 do.	.314538	-.006809	.321347
Wheat, standard middlings	Increase 1.0 do.	.151528	-.011615	.163143
Wheat, red dog flour	Increase 1.0 do.	.060017	.000242	.059775
Hominy feed, yellow	Increase 1.0 do.	.109861	.001952	.107909
Oats, excluding Pacific Coast	Increase 1.0 do.	.183536	-.012525	.196061
Oats, feeding, rolled	Increase 1.0 do.	.319169	.007058	.312111
Barley, excluding Pacific Coast	Increase 1.0 do.	.133136	-.011385	.144521
Tallow	Increase 1.0 do.	.119823	.079174	.040649
Bonemeal, steamed	Increase 1.0 do.	.802627	-.069912	.872539
Phosphate dicalcium	Increase 1.0 do.	1.053394	-.107281	1.160675
Phosphate defluorinated	Increase 1.0 do.	1.016187	-.078376	1.094563

exactly the specification for folic acid, even if no supplements had been added. In evaluating the ingredients for substitution in this formula, obviously no value should be placed on an increase in these four vitamins and no value should be placed on their reduction until such a reduction brings the vitamin level in the ingredients down to specification. Thus a more relevant comparison than that developed in table 14, so far as this formula is concerned, would have been one in which adjustment was made for changes in only pantothenic acid, niacin, and choline.

The values in table 13 permit the development of an adjustment factor for each ingredient for any of the many situations that may arise. The price of vitamin supplements changes infrequently; therefore, a set of adjustment factors in value terms for a given set of supplement prices could be prepared for use in adjusting the resulting net change in cost in table 4.

Ignoring the effect of change in vitamin content would not lead to serious error in choosing between ingredients, except possibly in the case of tallow and condensed fish solubles. Much of the vitamin value of dehydrated alfalfa meal is in the vitamin A content. As noted previously, it is doubtful whether vitamin A supplement is subject to economizing.

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